Shadow Pricing Wetland Function

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Abstract

This study uses the directional output distance function, a multi-output economic production frontier model, to estimate shadow prices for a set of non-marketed wetland ecological functions for the U.S. mid-Atlantic region Nanticoke River watershed. The estimation procedure adapts the bootstrap methods originally developed by Simar and Wilson (1998) for nonparametric efficiency estimates to the quadratic directional output distance function. The results from this application suggest that for some sites in this watershed, the value of improved wetland condition outweighs the potential value of agricultural production. On average, estimated values for improved wetland condition are also consistent with payments being made by the federal Wetlands Reserve Program (WRP) in the study area.

1 Introduction

Wetlands support a variety of human activities and provide critical habitat to numerous threatened and endangered species. In the lower 48 conterminous U.S. states, less than half of the estimated 220 million wetlands that existed

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prior to European settlement (Mitsch and Gosselink, 1986) remain today, and many of these are degraded (U.S. EPA, 2009).

Current U.S. environmental policy seeks to protect and recover wetlands areas, primarily through Section 404 of the 1977 Clean Water Act (CWA), which regulates the dumping of dredge and fill materials, by funding wetland restoration projects using the 1989 North American Wetlands Conservation Act (NAWCA), and by purchasing voluntary landuse easements through programs such as the USDA's Wetlands Reserve Program (WRP). Under these policies, the granting of pollution permits and the selection of projects for conservation funding rely on an implicit understanding of the associated costs and benefits of wetlands management to society, even when these values are not directly observed.

In this study, a set of wetland hydrological and biological functions are modeled jointly with agricultural output, as part of an economic production process to estimate the value of wetland condition at the watershed scale. We apply the directional output distance function (Chambers et al., 1996), a multi-output production frontier model, to the Nanticoke River watershed, which supports a significant and ecologically diverse system of wetland communities in the Mid-Atlantic region of the United States. Agricultural production accounts for much of the land use in the study watershed, and its associated drainage and channelization activities can significantly alter the hydrological function and biological integrity of surrounding wetland areas.

Wetland hydrological and biological function indices are combined with agricultural land values to estimate the value of individual wetland functions. In production theory, the directional output distance function is dual to the revenue function. One can exploit that duality to derive shadow price estimates for the hydrological and biological functions that support an array of wetland ecosystem services in the watershed. These services include flood control, water filtration, biodiversity,and riparian habitat. The results from this application suggest that for some sites in the watershed, the value of improved wetland condition outweighs the potential value of agricultural production. Conceivably, this price information could be used to prioritize wetland conservation efforts and influence land use policy in the watershed.

2 Valuing wetland services

The wetlands valuation literature is extensive. In a recent survey, Brander et al. (2006) refer to roughly 200 wetlands valuation studies, drawing on a range of valuation methods. Broadly, non-market valuation methods can be used to either directly or indirectly measure both the use and non-use value of ecosystem services (Hanley et al., 2007), where use implies a physical interaction with the environment (Barbier, 2007). For wetlands, use value can include flood-protection, water purification, recreation and aesthetic value, while non-use value might include the existence value of biodiversity and native vegetation.

Contingent valuation, the most common direct approach to valuation, uses stated preferences to estimate individuals' willingness to pay for non-marketed goods and services. Preferences are generally elicited through surveys or choice experiments, and can be used to estimate both use and non-use values. Notable examples of contingent valuation in the wetlands valuation literature include Hanneman et al. (1991), Loomis et al. (2000) and more recently, Birol et al. (2009). Heal et al. (2005) list two necessary conditions for contingent valuation: (1) the information must be available to describe the change in a natural ecosystem in terms of services that people care about, in order to place a value on those services and (2) the change in the natural ecosystem must be explained in the survey instrument in a manner that people will understand and not reject the valuation scenario. Often, in practice these conditions are difficult to meet, particularly as ecosystem complexity increases, which can bias the resulting estimates of willingness to pay.

Indirect approaches to valuation rely largely on revealed preferences, observed from behavior in related markets, to estimate the implicit value (as opposed to willingness to pay) of non-marketed goods and services. Hedonic pricing methods (Rosen, 1974) offer one such approach, by using market prices, most often for residential property, to infer value. Hedonic price methods model residential property as a composite good, composed of structural features (e.g., square footage, lot size), neighborhood characteristics (e.g., school quality, demographics), and environmental attributes (e.g., proximity to pollution, recreation opportunities). The property price, then, reflects the aggregate value of these individual characteristics, so that the marginal value of an individual characteristic can be derived through regression analysis.

Few hedonic pricing studies have been conducted to estimate the value of wetlands, perhaps due to the often rural location of wetland areas. Examples include Lupi et al. (1991), who estimate the value of wetland size, and Doss and Taff (1996), who estimate the value of proximity to different wetland types. Mahan et al. (2000) examine the effect of both wetland size and type on residential property values, and extend the literature by then using the implicit wetland prices from their hedonic model to estimate the willingness to pay for these attributes.

Other indirect approaches that have been used to value wetlands include the travel cost method (Cooper and Loomis, 1993), which bases value on the amount that individuals actually spend to visit a wetland (often for recreation activities such as bird-watching or hunting), and the estimation of replacement costs (Breaux et al., 1995; Byström, 2000), which measures the difference in the cost of using a wetland to provide a service (e.g., water purification) and the cost of using a man-made alternative (e.g., a water treatment facility).

Similar to hedonic methods, the production function approach to valuation models ecosystem services as environmental inputs (Mäler, 1992; Barbier, 2007) and then uses the market price of their associated outputs (e.g., agricultural products, fisheries) to derive input factor demands for these non-marketed resources. This draws on the earlier work of Becker (1965) and Lancaster (1966) to incorporate non-marketed resources such as time into the household production function. In this framework, household utility is a function of goods purchased for final use, as well as goods produced internally using both marketed and non-marketed inputs. Following this approach, one can derive the demand function for the non-marketed inputs by maximizing household utility subject to a budget constraint. This requires some understanding of the production function, namely the physical role of the environmental resource as an input. Applications in the wetlands literature include the valuation of mangroves as an input in coastal fisheries production (Barbier, 1994) and the the valuation of groundwater recharge as an input in agricultural production (Acharya, 2000; Koundouri and Xepapadeas, 2004).

Related to the production function approach, the frontier estimation methods developed for productivity and efficiency analysis offer another approach to non-market valuation. These include Shephard (1970) distance functions, hyperbolic distance functions (Färe et al., 1985) and directional distance functions (Chambers et al., 1996). Distance functions in general provide several advantages for environmental applications. They were originally developed for multi-input and multi-output production processes, and can thus accommodate multiple environmental attributes. Their estimation only requires quantity data for inputs and outputs, as opposed to price information, which enables the incorporation of non-marketed environmental goods and services into the production model. They also exhibit dual relationships to economic cost, revenue and profit functions, and it is this duality in production theory that underlies the use of distance functions as an approach to non-market valuation. Valuation applications using distance functions include the estimation of shadow prices for public land conservation (Färe et al., 2001); pollution (Färe et al.,1993; 2005; 2006; Coggins and Swinton, 1996; Ball et al., 2004; Murty et al., 2007); and, in the wetlands literature, groundwater recharge (Koundouri and Xepapadeas, 2004).

The present study further extends this approach to the wetlands literature, by using a directional output distance function to model the joint production of multiple wetland ecological functions and agricultural value within a watershed. The next section outlines the theory supporting this approach, followed by a discussion of estimation in practice. Section five presents an application to the Nanticoke River watershed in Delaware and an analysis of the results.

3 Underlying Theory

This study uses the directional output distance function, the production analogue to the Luenberger (1992) benefit function, to model the joint production of multiple wetland ecosystem services and agricultural value within a watershed area. Let $P(x)$ denote the feasible output set for the vector of outputs $y = (y_1, ..., y_M) \in \Re^M_+$ given inputs $x = (x_1, ..., x_N) \in \Re^N_+$, so that

$$P(x) = \{y : x \, \text{can produce} \, y\} \,. \tag{1}$$

In this context, outputs include a set of wetland function scores that are used to assess overall wetland condition, coupled with the value of nearby agricultural production. The wetland functions represent separate aspects of wetland condition, such as hydrological capacity and vegetation coverage. Watershed land area constitutes the shared input in this joint production process.

Following an axiomatic approach, a series of standard assumptions are made to characterize the production technology in theory, and to then guide the empirical specification of the model. The first of these assumptions is that $P(x)$ is convex and compact, so that the feasible output set is closed and bounded from above by the production frontier. Compactness acknowledges that output is scarce, by limiting the degree of wetland function and the level of agricultural production for a given land parcel, while convexity shapes the physical tradeoffs that occur along the production frontier. Secondly, outputs are assumed to be freely disposable, meaning that if $y \in P(x)$, then $y' \in P(x)$ for any $y' \leq y$. Free disposability allows any of the output levels to decrease without diminishing the prospects of other outputs. The impairment of one or more wetland functions does not necessarily reduce the potential of other wetland functions in the watershed, or the corresponding level of agricultural production. This also results in an implicit assumption of weak disposability, meaning that if $y \in P(x)$, then $\alpha y \in P(x)$ for any $0 \leq \alpha \leq 1$. It is possible to proportionally reduce any one of the outputs at no cost to the others.

Given these assumptions, the directional output distance function provides a complete representation of the feasible output set (Chambers et al., 1996), as well as individual measures of performance for each of the included output observations. The directional output distance function is defined as

$$\vec{D_O}(x,\, y; g_y) = max \left\{ \beta : [y + \beta g_y] \in P(x) \right\}, \tag{2}$$

where $g_y \in \Re^M_+$ is a directional vector that specifies the path of output expansion. This model measures each observation's distance, in a particular direction, to the production frontier. Thus, for observations on the frontier, $\vec{D_O}(x,\, y; g_y) = 0$, and for any observation below the frontier, $\vec{D_O}(x,\, y; g_y) > 0$. Individual performance deteriorates with distance to the frontier, so that the directional output distance value can be interpreted as a measure of inefficiency for each observation.

The directional output distance function offers a more general measure of productivity and efficiency than the more widely applied Shephard (1970) output distance function, which measures radial distance from the frontier and is defined as

$$D_O(x,\, y) = inf \left\{ \theta : (x,\, \frac{y}{\theta}) \in P(x) \right\}. \tag{3}$$

In fact, the Shephard output distance function can be constructed as a special case of the directional output distance function (Chambers et al., 1996) by specifying a radial direction vector, setting $g_y = y$. Figure 1 illustrates both the directional output distance function and the Shephard output distance function for $y = (y_1, y_2)$.

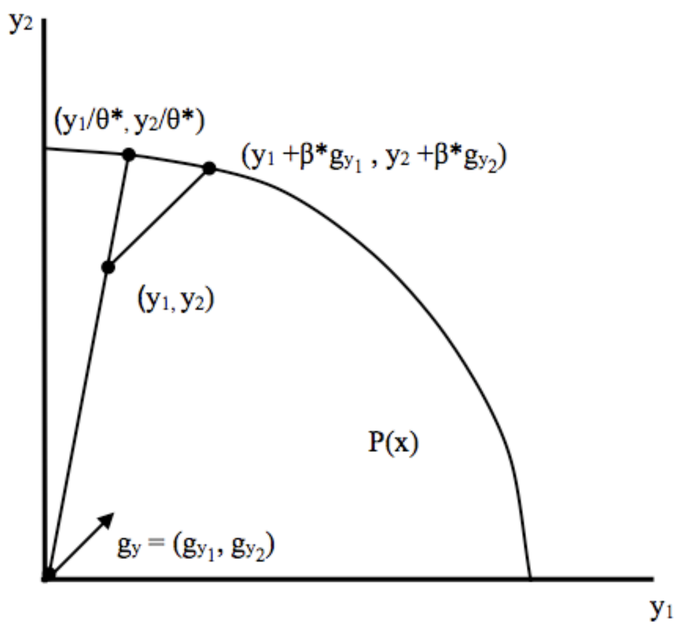

Figure 1: The directional and Shephard output distance functions

The directional output distance function accommodates both proportional and non-proportional output expansion, while the Shephard output distance function is restricted to a radial measure of efficiency. Drawing on the Shephard output distance function's homogeneity in y, for the case where $g_y = y$,

$$\vec{D_O}(x,\, y; g_y) = \frac{1}{D_O(x,\, y)} - 1. \tag{4}$$

The directional output distance function can also be used to account for the undesirable nature of some outputs of a production process, i.e., pollution, by specifying a negative direction for those outputs (Chung et al., 1997). This enables the simultaneous expansion of desirable output and contraction of undesirable output in the measurement of performance.The properties of the directional output distance function follow from the assumptions made to characterize $P(x)$, and include[1]

i. Representation

$\vec{D_O}(x,\, y; g_y) \geq 0$ if and only if $y \,\in\, P(x)$.

ii. Monotonicity

$\vec{D_O}(x,\, y'; g_y) \geq \vec{D_O}(x,\, y; g_y)$, for $y' \leq y$

iii. Translation

$\vec{D_O}(x,\, y + \alpha g_y; g_y) = \vec{D_O}(x,\, y; g_y) - \alpha$

iv. Directional Homogeneity of degree -1

$\vec{D_O}(x,\, y; \lambda g_y) = \lambda^{-1}\vec{D_O}(x,\, y; g_y)$, for $\lambda > 0$.

The directional output distance function fully represents the feasible output set. The first property states that the directional output distance must always be non-negative for any output vector within the feasible output set. The directional distance value is zero for output levels on the frontier and greater than zero for output levels below the frontier. Only infeasible output levels above the frontier would take a negative directional distance value, and any observed output vector, by existence, is feasible. The monotonicity property states that an increase in the output vector, holding inputs constant, can only reduce an observation's directional distance to the frontier, and thus can only improve the performance measure. The translation property states that the addition of any amount, α to an observed output vector in the direction

[1]Chambers et al. (1998) prove these properties for the input oriented case.

g_y reduces the directional distance value for that observation by the same amount, α. The directional output distance function provides an additive, as opposed to multiplicative, measure of distance to the frontier. The translation property serves as the additive analogue to the homogeneity property exhibited by the multiplicative Shephard distance function (Chambers et al., 1996). Related to this, the directional homogeneity property states that scaling the direction vector by any positive amount, λ proportionally reduces an observation's distance to the frontier by the same amount, λ.

The directional output distance function is used to construct the feasible output set for a vector of wetland functions and the value of agricultural production within a watershed area, which enables environmental performance assessment for each of the observation sites (Bellenger and Herlihy, 2009a; 2009b). Moreover, the resulting frontier reveals the physical tradeoffs that exist for the production of each of the wetland functions and agricultural output in the watershed. Given the market value of just one of the outputs, in this application, agricultural production, it is also possible to value these tradeoffs in monetary terms (Färe et al., 2001; 2005; 2006) by exploiting the directional output distance function's dual relationship to the revenue function,

$$R(x,\, p) = \max_{y} \left\{ py : y \in P(x) \right\}, \tag{5}$$

where $p = (p_1,\, ...,\, p_M) \in \Re^M_+$ is the vector of output prices corresponding to y. By definition,

$$R(x,\, p) \geq py, \forall y \in P(x), \tag{6}$$

and this, along with the definition of the directional output distance function from (2) and the representation property imply

$$
\begin{aligned}
R(x,\, p) &\geq p(y + \vec{D_O}(x,\, y; g_y)g_y) \\
&\geq py + \vec{D_O}(x,\, y; g_y)pg_y.
\end{aligned} \tag{7}
$$

Rearranging terms in (7),

$$
\vec{D_O}(x,\, y; g_y) \leq \frac{R(x,\, p) - py}{pg_y}. \tag{8}
$$

The directional output distance function can then be recovered from the right hand side in (8) as the solution to

$$
\vec{D_O}(x,\, y; g_y) = \underset{p}{min} \frac{R(x,\, p) - py}{pg_y}. \tag{9}
$$

The vector of shadow prices, p is derived by applying the envelope theorem to (9), so that

$$
\nabla_y \vec{D_O}(x,\, y; g_y) = \frac{-p}{pg_y} \leq 0, \tag{10}
$$

and for a single observation,

$$
\frac{p_m}{p_{m'}} = \frac{\partial \vec{D_O}(x,\, y; g_y)/\partial y_m}{\partial \vec{D_O}(x,\, y; g_y)/\partial y_{m'}}, \forall m, m' \in M. \tag{11}
$$

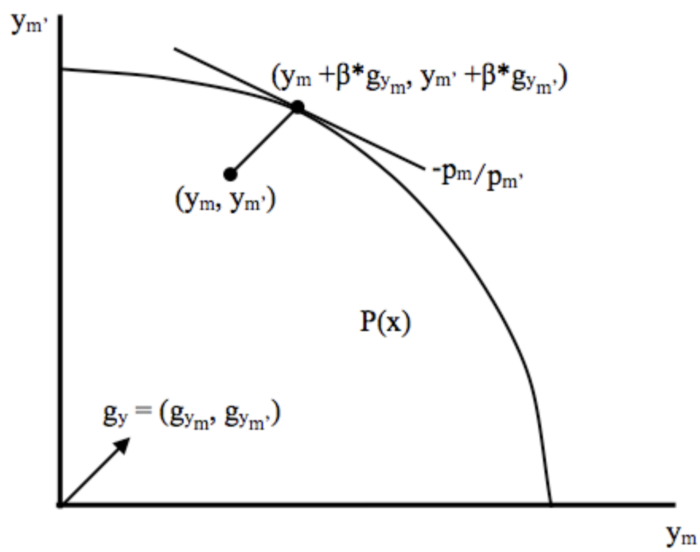

Figure 2: Directional output distance and shadow prices

The shadow price ratio in (11) reflects the relative value of each output, which equals the corresponding marginal performance[2] ratio, as depicted in Figure 2. If at least one of the outputs in $P(x)$ is marketed, then it is also possible to recover the real, as opposed to relative, value of the non-marketed outputs (Färe et al., 2001). For example, if the m'^{th} attribute is actually a marketed good, and assuming its shadow price is equal to its market price, then the price of the m^{th} attribute can be recovered as

$$p_m = p_{m'} \frac{\partial \vec{D_O}(x,\, y; g_y)/\partial y_m}{\partial \vec{D_O}(x,\, y; g_y)/\partial y_{m'}}. \tag{12}$$

Given the marketed nature of agricultural output, this study uses (12)

[2]This term refers to the marginal productivity of each wetland function. Recall, the directional output distance function provides a measure of environmental performance for each observation, so that marginal performance reflects the degree to which an increase in each of the wetland functions reduces an observation's directional distance to the output frontier, i.e. improves performance.

to estimate the value of each of the non-marketed (but arguably valuable) wetland functions. The resulting price vector captures in some sense the implicit value of these wetland functions, by valuing marginal productivity in terms of the corresponding value of agricultural production, but is not analogous to implicit value in the spirit of hedonic analysis. In this case the price vector represents the value of the tradeoff between agricultural production and each of these wetland functions in the watershed area, rather than the value of each wetland function's contribution to agricultural output, as in hedonic analysis.

4 Estimation

One practical advantage afforded by the directional output distance function lies in the flexibility of its estimation. Depending on the research objective, one can choose to estimate this model either parametrically or nonparametrically. Nonparametric estimation relies on data envelopment analysis (DEA)(Charnes et al., 1978) methods to construct the feasible output set as a convex, linear combination of all input and output observations. The resulting model satisfies the assumptions made to characterize $P(x)$ and assesses performance by measuring each observation's directional distance to the corresponding output frontier, a piece-wise linear combination of the outermost output observations. DEA estimation does not, however, generate the smooth, differentiable output frontier required to solve for unique shadow values, as outlined in the previous section, and thus does not offer a tractable way to evaluate the economic tradeoffs facing each of the observations[3].

Hence, parametric estimation is employed to construct a differentiable output frontier which, via duality, can then be used to value each of the non-marketed wetland outputs following (12). Parameterization must satisfy the axiomatic properties of the directional output distance function and enable

[3]Chambers and Färe (2008) offer one exception.

the computation of marginal effects. In addition, linearity (in the parameters) facilitates estimation. This limits the set of possible functional forms considerably. To guide the choice of functional form in estimating production frontiers, Chambers (1988) explains, "...the primary goal of applied production analysis is empirical measurement of the economically relevant information that exhaustively characterizes the behavior of economic agents. For smooth technologies (i.e., those that are twice-continuously differentiable), this includes the value of the function (e.g., the level of cost), the gradient of the function (e.g., the derived demands), and the Hessian (e.g., the matrix of derived-demand elasticities)."

It is possible to approximate any arbitrary smooth function with a linear function, meaning that the parameters of the linear function can be restricted so that the linear function value, gradient, and Hessian are equal in magnitude to the corresponding values of the smooth function evaluated at a particular point on its domain. Such linear functions are known as *second-order differential approximations* (Chambers, 1988). It is also possible to restrict the parameters of a linear function to provide a Taylor's series approximation of an arbitrary smooth function. Such linear functions are known as *second-order numerical approximations* (Chambers, 1988). To be considered *flexible*, a functional form must either provide a second-order differential approximation or second-order numerical approximation (Chambers, 1988).

For an arbitrary smooth function $F: R^L \to R$, the second-order Taylor's series approximation of $F(q)$ evaluated at $q^* \in R^L$ is

$$F(q) = F(q^* + DF(q^*)(q - q^*) + \frac{1}{2}(q - q^*)D^2F(q^*)(q - q^*), \tag{13}$$

where $DF(q^*)$ is the Jacobian matrix of $F(q)$ at q^*. The generalized quadratic (Chambers, 1988), also known as a 'transformed quadratic function' (Diewert, 2002) or a 'second-order Taylor series approximation interpretation function'(Färe and Sung,1986)(Färe et al., 2008), can be restricted to represent the second-order Taylor's series approximation of an arbitrary twice-continuously differentiable function. This is given as

$$G[F(q)] = a_0 + \sum_{l=1}^{L} a_l h(q_l) + \sum_{l=1}^{L}\sum_{l'=1}^{L} a_{ll'} h(q_l) h(q_{l'}), \tag{14}$$

where $h : R \to R$ is twice differentiable and G is invertible. The generalized quadratic encompasses a set of flexible functional forms, flexible in both the differential and numerical sense (Chambers, 1988). Of these, just two are known to satisfy (with parameter restrictions) the translation property of the directional output distance function (Färe and Lundberg, 2006). These are the quadratic function

$$F(q) = a_0 + \sum_{l=1}^{L} a_l q_l + \sum_{l=1}^{L}\sum_{l'=1}^{L} a_{ll'} q_l q_{l'}, \tag{15}$$

and the unnamed function

$$F(q) = \frac{1}{2\lambda} ln \sum_{l=1}^{L}\sum_{l'=1}^{L} a_{ll'} \exp(q_l) \exp(q_{l'}). \tag{16}$$

The second of these, however, does not contain the first order parameters needed to compute marginal effects. This leaves the quadratic form as the only known flexible functional form that can be restricted to satisfy the translation property. Most recently, Färe et al. (2010) use Monte Carlo simulations to demonstrate the apparent greater ability in practice of the quadratic directional output distance function, compared to the translog (also flexible and can be likewise restricted to satisfy homogeneity) Shephard output distance function to characterize the output set. The quadratic (also as in Aigner and Chu, 1968) directional output distance function(Färe et al., 2001; 2005; 2006) for $g_{y_m} = 1$, $m = 1,...,M$ is estimated as

$$\vec{D_O}(x,\ y; g_y) = \alpha_0 + \sum_{n=1}^{N} \alpha_n x_n + \sum_{m=1}^{M} \beta_m y_m + \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \alpha_{nn'} x_n x_{n'} + \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \beta_{mm'} y_m y_{m'} + \sum_{n=1}^{N}\sum_{m=1}^{M} \gamma_{nm} x_n y_m. \tag{17}$$

To satisfy the translation property, set

$$\sum_{m=1}^{M} \beta_m g_{y_m} = -1;$$
$$\sum_{m=1}^{M} \gamma_{nm} g_{y_m} = 0, n = 1, ..., N;$$
$$\sum_{m'=1}^{M} \beta_{mm'} g_{y_{m'}} = 0, m = 1, ..., M,$$

and to ensure symmetric cross-input and cross-output effects, set

$$\alpha_{nn'} = \alpha_{n'n}, n, n' = 1, ..., N;$$
$$\beta_{mm'} = \beta_{m'm}, m, m' = 1, ..., M,$$

Given the quadratic form, the marginal effect of the m^{th} attribute is then derived as

$$\frac{\partial \vec{D_O}(x,\ y; g_y)}{\partial y_m} = \beta_m + \sum_{m'=1}^{M} \beta_{mm'} y_{m'} + \sum_{n=1}^{N}\sum_{m=1}^{M} \gamma_{nm} x_n, \tag{18}$$

and the shadow price ratio for the m^{th} and m'^{th} attributes is written as

$$\frac{p_m}{p_{m'}} = \frac{\beta_m + \sum\limits_{m'=1}^{M} \beta_{mm'} y_{m'} + \sum\limits_{n=1}^{N} \sum\limits_{m=1}^{M} \gamma_{nm} x_n}{\beta_{m'} + \sum\limits_{m=1}^{M} \beta_{mm'} y_m + \sum\limits_{n=1}^{N} \sum\limits_{m'=1}^{M} \gamma_{nm'} x_n}. \tag{19}$$

One can estimate the quadratic directional output distance function as a constrained linear programming problem, choosing the parameters to minimize each observation's distance to the frontier[4]. The solution to this problem, the optimal values for $\alpha_0, \alpha_n, \beta_m, \alpha_{nn'}, \beta_{mm'}, \gamma_{nm}$, and D_O^k minimize

$$\sum_{k=1}^{K} \vec{D_O}^k (x^k,\, y^k; g_y) \tag{20}$$

subject to

i. Representation

$$\vec{D_O}^k (x^k,\, y^k; g_y) \geq 0, k = 1, ..., K,$$

ii. Monotonicity

$$\frac{\partial \vec{D_O}^k (x^k,\, y^k; g_y)}{\partial y_m^k} \leq 0, m = 1, ..., M, k = 1, ..., K,$$

$$\frac{\partial \vec{D_O}^k (x^k,\, y^k; g_y)}{\partial x_n^k} \geq 0, n = 1, ..., N, k = 1, ..., K,$$

[4] The quadratic directional output distance function can also be estimated as a stochastic frontier (Färe et al., 2005), following stochastic frontier methods outlined in Kumbhakar and Lovell (2000).

iii. Translation

$$\sum_{m=1}^{M}\beta_m g_{y_m} = -1; \sum_{m=1}^{M}\gamma_{nm} g_{y_m} = 0,\ n = 1, ..., N;$$

$$\sum_{m'=1}^{M}\beta_{mm'} g_{y_{m'}} = 0,\ m = 1, ..., M,$$

iv. Symmetry

$$\alpha_{nn'} = \alpha_{n'n},\ n, n' = 1, ..., N;$$

$$\beta_{mm'} = \beta_{m'm},\ m, m' = 1, ..., M.$$

The constraints ensure that the quadratic form satisfies the properties of the directional output distance function. The first constraint satisfies the representation property by requiring all observations to either lie on or below the output frontier. The second constraint states that an increase in any output, or a decrease in any input can only reduce an observation's distance to the output frontier, which guarantees monotonicity for both inputs and outputs. The third constraint imposes the translation property, restricting the parameters so that additional output in the g_{y_m} direction reduces an observation's distance to the frontier by an equal amount. The final constraint restates the symmetry condition for cross-input and cross-output effects.

The resulting parameter estimates, and corresponding distance and shadow price values are based on an estimate of the true, but unobservable output frontier. This estimated frontier depends on the given sample of observations, rather than any *a priori* knowledge of the true frontier. Sampling variation, then, would potentially not only alter the estimated frontier, but would also affect the individual performance measures by changing each observation's proximity to the frontier.

Simar and Wilson (1998) introduce the bootstrap (Efron, 1979) as a way to analyze the sensitivity of DEA estimates to sampling variation, through repeated sampling. A similar approach is applied to the directional output distance function herein, to better understand the statistical properties of the distance and shadow price estimates. This method solves (16) for repeated samples, each drawn with replacement from the original data sample. This yields new parameter estimates for each simulated sample, which are then used to construct distance and shadow price values for the original data, thus providing a known distribution of distance and shadow price estimates.

To explain, let $\hat{\theta}$ be the vector of parameter estimates that solve (16) to give $\hat{\vec{D_O}}$ and $\hat{p_m}$ for the original data, (x, y), where (x, y) constitutes a random sample generated by an unknown process, F, the true DGP. Without knowing the true DGP, this approach instead uses an estimate of the DGP, $\hat{F}$, based on the empirical distribution to generate S bootstrap samples, (x_s^*, y_s^*), $s = 1, ..., S$. For the s^{th} iteration, each k^{th} observation from the original sample is drawn (with replacement) with probability $\frac{1}{k}$. This results in a new pseudo feasible output set, $P(x_s^*)$. Applying (16) to (x_s^*, y_s^*) then generates the vector of parameter estimates, $\hat{\theta}_s^*$, which are used to compute the bootstrapped directional output distance, $\vec{D_O}_s^k$ for the original data, (x, y). The bootstrapped directional output distance function is defined for the s^{th} iteration as

$$\vec{D_O}_s^*(x,\, y; g_y) = max\left\{\beta^* : [y + \beta^* g_y] \in P(x_s^*)\right\}, \tag{21}$$

and the corresponding shadow price ratio for the m^{th} and m'^{th} attributes is written as

$$\frac{p_{m_s}^*}{p_{m'_s}^*} = \frac{\beta_{m_s}^* + \sum\limits_{m'=1}^{M} \beta_{mm'_s}^* y_{m'} + \sum\limits_{n=1}^{N}\sum\limits_{m=1}^{M} \gamma_{nm_s}^* x_n}{\beta_{m'_s}^* + \sum\limits_{m=1}^{M} \beta_{mm'_s}^* y_m + \sum\limits_{n=1}^{N}\sum\limits_{m'=1}^{M} \gamma_{nm'_s}^* x_n}. \tag{22}$$

This approach raises one immediate concern. If a bootstrap data sample, (x_s^*, y_s^*) does not include the observations lying on or near the frontier for the original data, then this approach could potentially render negative directional distance values for the highest performing observations from the original data. As a simple remedy, the proposed method discards any iteration that yields a negative distance value[5]. The rationale follows from the fact that the estimated frontier for the original data can never surpass the true frontier. This biases the distance estimates downward, and retaining iterations that omit the highest performing sites only furthers that downward bias.

As long as the empirical distribution adequately approximates the true DGP, then the known bootstrapped distribution of parameter estimates should closely resemble the true, but unknown, parameter distributions, so that

$$(\hat{\theta}^{*} - \hat{\theta})|\hat{F} \sim (\hat{\theta} - \theta)|F \tag{23}$$

Given (19), this approach uses the bootstrapped distance and shadow price distributions to estimate the bias and standard error for the original distance and shadow price estimates. Following Simar and Wilson (1998), for each observation the original distance estimate bias,

$$\text{bias}_{F,k} = E_F(\hat{\vec{D}}_O^k) - \vec{D}_O^{\,k} \tag{24}$$

is estimated using its bootstrap estimate

$$\text{bias}_{\hat{F},k} = E_{\hat{F}}(\vec{D}_O^{\,k^*}) - \hat{\vec{D}}_O^k, \tag{25}$$

which is approximated by

$$\hat{\text{bias}}_{\hat{F},k} = \frac{1}{S}\sum_{s=1}^{S} \vec{D}_{O\,s}^{\,k^*} - \hat{\vec{D}}_O^k = \bar{\vec{D}}_O^{k^*} - \hat{\vec{D}}_O^k. \tag{26}$$

[5] In this empirical application, less than five percent of iterations were discarded due to negative distance values.

The bias-corrected distance estimate, $\tilde{\vec{D}}^k_O$ is defined as

$$\tilde{\vec{D}}^k_O = \hat{\vec{D}}^k_O - \hat{\text{bias}}_{\hat{F},k} = 2\hat{\vec{D}}^k_O - \bar{\vec{D}}^{k^*}_O, \tag{27}$$

and the estimated standard error of $\hat{\vec{D}}^k_O$ is

$$\hat{\text{se}} = \left[\frac{1}{S-1} \sum_{s=1}^{S} ({\vec{D}_{O_s}}^{k^*} - \bar{\vec{D}}^{k^*}_O)^2 \right]^{\frac{1}{2}}. \tag{28}$$

A similar argument applies to the shadow price estimates.

5 Empirical Application

To illustrate the directional distance approach to shadow pricing, a set of wetland hydrogeomorphic (HGM) indicator data is combined with agricultural land values for the Nanticoke River watershed, which constitutes "one of the most biologically important and wetland-rich watersheds in the [United States] mid-Atlantic region(The Nature Conservancy, 1994; Whigham et al., 2007)." Figure 3 depicts the watershed area[6] for the Nanticoke river, which flows into Chesapeake Bay, and drains roughly 283,000 ha in Maryland and Delaware (Jacobs et al., 2010). The application in this study examines the Delaware portion of the watershed. The HGM indicator data was originally collected to asses wetland condition in the Nanticoke watershed[7] as part of the U.S. Environmental Protection Agency's (EPA) Regional Environmental Monitoring and Assessment Program (REMAP). The HGM data includes both physical (e.g., drainage, sedimentation and floodplain conditions) and biological (e.g., vegetation composition and density) indicators of condition for riverine and flats wetland classes within the Nanticoke watershed. The REMAP Nanticoke assessment collected more than 20 separate wetland in-

[6] The map source is Jacobs et al., 2007.

[7] For a detailed discussion of the wetland indicator data, refer to Whigham et al., 2007.

dicators, which they then used to construct functional capacity index (FCI) scores (ranging in value from 0 to 1) for a set of wetland functions. These function scores serve as outputs in this application, and the directional output distance function is used to estimate their respective shadow prices. The functions (and their associated indicators) are:

i. Hydrology (sedimentation, drainage, and floodplain conditions)

ii. Biogeochemistry (topographic features, vegetation density and composition, Hydrology)

iii. Vegetation (vegetation density and composition, invasive species)

iv. Habitat (vegetation disturbance, coverage, and density, onsite stream condition)

v. Buffer (surrounding vegetation and proximal stream condition)

These functions directly relate to many of the ecosystem services commonly attributed to wetlands. For instance, the hydrology function score provides a measure of flood control capacity. The biogeochemistry function score reflects the area's water filtration potential. Vegetation condition contributes to biodiversity and carbon sequestration. Riparian habitat supports all manner of wildlife in the watershed, which fosters recreational hunting and fishing benefits. The buffer function measures the surrounding wetland condition, and may capture some of the scenic or aesthetic value of the watershed.

The biogeochemistry function is omitted from analysis, as it is constructed as a linear combination of the hydrology function. The remaining functions do not share individual indicators, i.e. the vegetation function uses a different vegetation density indicator than the vegetation density indicator that is used to construct the habitat function score. Table 1 describes the wetland function scores, which range from 0.01 to 1 in order of improved condition,

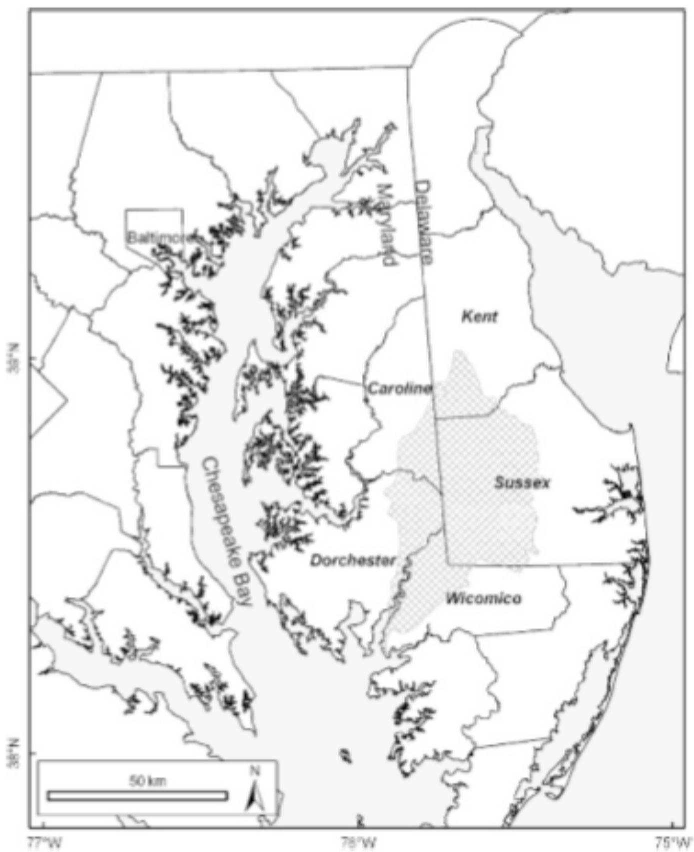

Figure 3: The Nanticoke watershed (hatched area)

for 86 riverine (29 obs.) and flats (57 obs.) observations. In a related study (Jacobs et al., 2010) the HGM indicators are also used to construct an overall index of wetland condition (IWC) for riverine, flats and depression land class areas within the Nanticoke watershed. The IWC (also listed in Table 1) can range in value from 0 to 100, in order of improving condition, and in this application ranges from 22 to 100.

The shadow pricing methodology outlined in the previous sections is applied to the wetland function scores, coupled with data on the value of agricultural production in the watershed area to better understand the economic value (in terms of opportunity cost) of these functions. Over half of all land in the watershed is devoted to agricultural use, and farmers in the study area use more than 80 percent of their land to produce corn, wheat and soybean crops (USDA NRCS, 2007). To capture the potential value of agricultural production in the watershed area, this study uses agricultural land value, which in an efficient market should equal the discounted stream of agricultural production value for a given parcel of land.

The application draws on agricultural land values from a data set compiled by the Delaware Agricultural Lands Preservation Program[8]. This program has two stated goals: i.) to preserve a critical mass of crop land, forest land, and open space to sustain Delaware's agricultural industry, and ii.) to provide landowners an opportunity to preserve their land in the face of increasing development pressures and decreasing commodity values. Notably, this is not a wetlands preservation program. Instead, the program seeks to maintain land in agricultural production and to prevent commercial or residential development on agricultural lands.

Landowner participation in the agricultural preservation program is voluntary and competitive. To join, a landowner or group of owners must first form an Agricultural Preservation District, comprised of at least 200 con-

[8]For additional information, refer to the program website: http://dda.delaware.gov/aglands/lndpres.shtml

tiguous acres already devoted to agricultural activities. Landowners within the district agree not to develop their land for at least 10 years and are compensated with tax benefits and 'right-to-farm' protection (protection from nuisance lawsuits filed by non-farming neighbors) for participating in the program. More importantly, initial participation also qualifies the landowner to apply for a permanent easement, or the sale of all future development rights for their land. To date, agricultural landowners have formed over 500 preservation districts, containing roughly 130,000 acres. Half of this land, approximately 65,000 acres, is protected by permanent development easements.

The program assesses both the fair-market value, which includes potential development value, and the agricultural value of lands enrolled in permanent easement, and generally offers subsequent easement payments between these two values. For the study area easement parcels, the average fair market value, agricultural value, and easement payment are, respectively, $US 3,844/acre, $US 1,440/acre and $US 1,833/acre. The program does not release land value assessments, either fair market or agricultural, for land in temporary easement, which limits the present analysis of agricultural value to a set of 29 agricultural land parcels in permanent easement, averaging 149 tillable acres in size. This does, however, avoid multiple time horizons, as well as the inclusion of possible future development value.

The agricultural land parcels with permanent development easements are used to construct a spatially-weighted agricultural land value for each wetland observation site, based on the assessed agricultural values of the five closest easement parcels. The study area is relatively small, and generally, these agricultural parcels are quite close, averaging less than five miles from their corresponding wetland observation sites. The resulting distance-weighted agricultural value per acre for the k^{th} observation, AV^k, is

$$AV^k = \frac{\sum_{i=1}^{5} Dist_{ik} \times AV_i}{\sum_{i=1}^{5} Dist_{ik}}, \tag{29}$$

where $Dist_{ik}$ is the Euclidean distance from the boundary of the i^{th} easement parcel to the k^{th} wetland observation, and AV_i is the assessed agricultural value for easement parcel i. This results in an average agricultural value of $ US 1,572/acre[9] for the wetland observation sites. Table 1 provides descriptive statistics for the constructed agricultural values.

Table 1: Descriptive Statistics for the Nanticoke Data Set (86 Obs.)

Wetland Service	Mean	Std. Dev.	Min.	Max.
Hydrological Function	0.593	0.368	0.011	1.000
Vegetation Function	0.767	0.229	0.150	1.000
Habitat Function	0.659	0.252	0.100	1.000
Buffer Function	0.816	0.193	0.125	1.000
Agriculutral Value ($1,000/ acre)	1.572	0.391	1.078	3.725
Index of Wetland Condition (IWC)	72.017	19.492	22.417	99.833

The wetland function scores can range in value from 0.010 to 1.000, and on average, each of the functions in the study area are roughly 0.60 or higher. Given the relatively small number of observations, 29 classified as riverine (primarily riparian and floodplain) and 57 classified as flats (primarily floodplain) [10], and the similarity of these two land classes, this study combines the riverine and flats observations into one data set for estimation. Typically, observation sites in both classes have lower hydrological function and higher buffer function values in the study area. The overall index of wetland condition can range from 0 to 100, and is roughly 72.0 on average[11]. The

[9] All land values are represented in $US 2009.

[10] Refer to Brinson (1993) for HGM wetlands classification criteria.

[11] For construction of the IWC, refer to Jacobs et al. (2010).

spatially-weighted agricultural land values on nearby farms average roughly \$1,572 per acre.

The application models each of the wetland function scores, along with the value of agricultural production, as outputs that can be jointly produced on land in the study area. For computational purposes, each of the observed function scores and agricultural land values are divided by their respective sample mean, so that $y_m = 1, m = 1, ..., 5$ for a hypothetical observation at the mean. This deflation ensures independence of unit of measurement (Shephard, 1970), and corrects for differences in scale, due to the differences in unit of measurement between the function scores and the agricultural land values. The shared input in this application is land, normalized to one acre, so that wetland function and agricultural land value are measured on the scale of one acre of land. For estimation purposes, a constant input (in this case, one acre of land) is equivalent to modeling production without inputs (Lovell and Pastor, 1997). The resulting parameter values are listed in Table 2.

The bootstrap methods outlined in Section 3 are also applied to better understand the sensitivity of these parameter values, as well as the distance and shadow price estimates, to sample variation. Thus, each observation's distance is solved repeatedly, as explained in (21) for 500 samples drawn, with replacement, from the original data set. This yields a distribution of sample parameter values, along with a distribution of distance and shadow price estimates for each observation. These distributions are then used to compute the standard error for each parameter, distance, and shadow price estimate, and to then correct those estimates for bias as outlined in Section 3. The standard errors and bias-corrected parameter values are listed in Table 2. Table 3 reports the bias-corrected distance and shadow price ratio estimates.

Given the mean weighting normalization employed for computation, the distance value for a hypothetical observation at the mean can be interpreted as the percent increase in output value y_m^k required to reach the corresponding frontier value y_m^{k*}. This relationship is independent of unit of measurement, and is written for each observation k as

$$\frac{y_m^k}{\bar{y}_m} + \vec{D_O}^k = \frac{y_m^{k*}}{\bar{y}_m}, \tag{30}$$

which implies

$$y_m^k + \vec{D_O}^k \bar{y} = y_m^{k*}. \tag{31}$$

Using this relationship, the results indicate that observations within the entire sample can, on average, increase each of their metric levels by roughly 19 percent of the respective output mean. The hydrology function, for example, has a mean value of 0.593. This means that on average, observations in the sample could add $0.194 \times 0.593 = 0.115$ to their hydrology function score.

Table 2: Bootstrap Parameter Estimates

Coefficient	Variable	Deterministic	Standard Error	Bias-Corrected
α_0	Constant	1.221	0.002	1.220
β_1	y_1	-0.230	0.009	-0.021
β_2	y_2	-0.320	0.041	-0.343
β_3	y_3	-0.037	0.014	-0.033
β_4	y_4	-0.536	0.029	-0.527
β_5	y_5	-0.084	0.018	-0.076
β_{11}	y_1^2	-0.009	0.004	-0.008
β_{21}	y_2y_1	0.013	0.010	0.015
β_{22}	y_2^2	-0.159	0.037	-0.151
β_{31}	y_3y_1	0.000	0.004	-0.001
β_{32}	y_3y_2	-0.016	0.011	-0.022
β_{33}	y_3^2	-0.007	0.013	0.001
β_{41}	y_4y_1	-0.005	0.009	-0.008
β_{42}	y_4y_2	0.168	0.044	0.176
β_{43}	y_4y_3	0.018	0.015	0.014
β_{44}	y_4^2	-0.182	0.068	-0.190
β_{51}	y_5y_1	0.001	0.006	0.002
β_{52}	y_5y_2	-0.006	0.036	-0.018
β_{53}	y_5y_3	0.006	0.009	0.008
β_{54}	y_5y_4	0.001	0.030	0.008
β_{55}	y_5^2	-0.002	0.011	0.000

For each wetland function, the shadow price ratio is equal to the ratio of that function's marginal performance to the marginal performance of agricultural value, where marginal performance refers to each output's marginal contribution to reducing distance to the frontier. This ratio reflects the relative value of each wetland function to the value of agricultural output, at each observation site. In this application, relative values may provide a more intuitive representation of the value of each of these functions. To explain, the absolute shadow price, p_m, for each function is computed from that function's ratio of marginal performance to agricultural value ($y_{m'}$). Given the mean-weighting procedure employed for computation, this means that for a given observation,

$$p_m = p_{m'} \frac{\partial \vec{D_O}(x,\, y; g_y)/\partial y_m}{\partial \vec{D_O}(x,\, y; g_y)/\partial y_{m'}} \frac{\bar{y_{m'}}}{\bar{y_m}}, \tag{32}$$

where $\bar{y_m}$ refers to the sample mean for wetland function y_m and $\bar{y_{m'}}$ refers to the sample mean agricultural value per acre. Agricultural land value is measured in dollars per acre, so that $p_{m'}$, the value of an additional dollar in value per acre, is equal to $1. Again, using the hydrology function as an example, this translates on average to an absolute shadow price of $812, or $81.24 for a 0.1 increase in hydrological function score. This illustrates a second challenge to understanding the value of non-marketed environmental attributes, such as wetland function. In addition to the lack of market price information, there is often a lack of 'market units'. This application values the marginal increase in wetland function score. For this to be more tangible in practice, it is important to consider the physical implications of that increase in function score, to better understand the actual improvement being valued. Table 6 presents the function shadow prices.

Table 3: Bias-Corrected Distance and Shadow Price Ratios* (86 Obs.)

Variable	Mean	Std. Dev.	Min	Max
Distance	0.194	0.173	-0.003	0.862
Hydrological Function	0.307	0.219	0.000	0.812
Vegetation Function	4.420	1.228	0.681	7.764
Habitat Function	0.428	0.144	0.000	0.661
Buffer Function	7.299	2.873	1.101	18.651

*Note, the shadow price ratios are relative to agricultural land value.

A separate model also estimates the directional output distance function and shadow price ratio, as above, for the overall index of wetland condition. Recall, the IWC includes many of the same variables that are used to construct the individual function scores, but is not simply a function (e.g. linear combination, geometric mean, etc.) of the individual wetland function scores. The same bootstrap procedure is applied. The IWC model parameter values are listed in Table 4 and the distance and shadow price values are listed in Table 5. The Spearman's rank correlation between the IWC distance values and those of the previous model is 0.71. These results suggest that on average, a hypothetical observation at the mean can increase its IWC by 37 percent, which translates to an IWC increase of 26.65. Because the second order parameter values, β_{21} and β_{22} are zero, all observations have the same shadow price ratio (see equation (19)) of 8.491. This also implies that each of the observations has the same IWC shadow price of \$185 for a 1 point increase in IWC value.

Table 4: IWC Bootstrap Parameter Estimates

Coefficient	Variable	Deterministic	Standard Error	Bias-Corrected
α_0	Constant	1.371	0.000	1.371
β_1	y_1	-0.894	0.000	-0.895
β_2	y_2	-0.105	0.000	-0.105
β_{11}	y_1^2	0.000	0.000	0.000
β_{21}	$y_2 y_1$	0.000	0.000	0.000
β_{22}	y_2^2	0.000	0.000	0.000

Table 5: Bias-Corrected Distance and IWC Shadow Price Ratio

Variable	Mean	Std. Dev.	Min	Max
Distance	0.371	0.246	0	1.008
IWC	8.491	0.000	8.491	8.491

Table 6: Bias-Corrected Shadow Prices

Variable	Mean	Std. Dev.	Min	Max
Hydrological Function	81.24	58.03	0.00	215.31
Vegetation Function	905.71	251.56	139.59	1,591.08
Habitat Function	102.14	34.46	0.00	157.60
Buffer Function	1,405.99	553.46	212.00	3,592.51
IWC	185.31	0.00	185.31	185.31

The absolute shadow prices[12] for each of the wetland functions, as well as the IWC are listed in Table 6. One can use these shadow prices to estimate the value of improved wetland function for each observation, by valuing the maximal expansion of wetland function. For instance, consider a hypothetical observation with mean distance and shadow price values, which implies a maximal expansion of roughly 19 percent for each function score. Again, for the hydrological function, this translates to an increase in function score of .115. The value of that increase in hydrological function on one acre, using the mean shadow price, is \$US 93.43 (\$US 81.24 $\times$ 1.15). Given the shadow prices for each of the wetland functions, a 19 percent increase in all wetland functions translates to a value of \$US 2,995/acre for this hypothetical observation with mean distance and shadow price values. Applying this approach to each observation in the watershed results in an average value of $\$US3,830$/acre for maximal improved wetland function. Because the shadow prices are functions of agricultural land value, which captures the discounted stream of agricultural production value, this amount reflects the discounted stream of function value, as opposed to annual value. Or, in other words, this is the amount, in terms of foregone agricultural value, that could be sacrificed to support the maximal increase in function value for each observation.

[12]Note the wetland function shadow prices represent the value of a 0.1 unit increase in the corresponding function value. The IWC shadow price refers to a 1 unit increase in the IWC value. Recall, each of the function scores can range from 0 to 1 and the IWC can range from 0 to 100.

Turning to the IWC, recall that the distance results imply that a hypothetical observation at the mean could increase its IWC value by 37 percent, or 26.65 points. The estimated value of that increase, using the mean shadow price, is roughly $US 4,939/acre. Applying this approach to observations in the watershed results in an average value of $US 4,957/acre for maximal improved wetland condition. This is higher, but not surprising given that the IWC places more weight on some of the vegetation indicators that were used to construct the higher-valued vegetation function[13].

The average value for function improvement ($US 3,830/acre) exceeds the average assessed agricultural value ($US 1,440/acre) and nearly equals the average assessed market value ($US 3,844/acre) for land in the study area. This suggests that the benefits of improving wetland function may outweigh the value of agricultural production, and the average amount ($US 1,833/acre) spent to maintain land in agricultural production by the agricultural preservation program, for many of the wetland sites in this study. The average value for IWC improvement ($US 4,957) outweighs the average assessed market value for land in the study area, suggesting that for many of the wetland sites in this study, the value of improved wetland condition potentially outweighs development value. These estimated values are fairly consistent with levels that are actually being spent to protect wetlands in the area. The WRP Delaware permanent[14] easement rates for 2009 are $US 2,900/acre for woodland and $US 4,000/acre for cropland (USDA, 2008), in addition to 100 percent of wetlands restoration costs.

It is also important to consider how these prices compare to other estimated wetlands values in the literature. In their meta-analysis of the literature, Brander et al. (2006) estimate an average annual value of roughly $US 1,134/acre, based on 80 reasonably comparable studies. There is, however, no differentiation in their estimate for wetland type or quality.

[13]Refer to Jacobs et al., 2010 for further discussion of the IWC.

[14]The 30-year easement rate is 75 percent of permanent easement values.

Despite the large number of existing wetlands valuation studies, relatively few value wetland function. Within these, value estimates and application settings range widely. Ragkos et al. (2006) connect a set of wetland functions (e.g., groundwater recharge, floodwater retention, sediment retention) for the Zazari-Cheimaditida wetland in Greece to various wetlands services (e.g., irrigation water, flood control, fisheries), and then use contingent valuation estimates for those services to value the corresponding functions. They find that on average, individuals in the area are willing to pay 40-43 Euros (roughly $US 50-54) annually to maintain each of the wetland functions. Acharya (2000) uses the production function approach (see Section 2) to estimate a total value of $US 13,029 per day for hydrological recharge in Nigeria's Hadejia-Nguru wetlands. Using contingent valuation, Loomis et al. (2000) estimate an annual willingness to pay of $US 19-70 million for the restoration of ecosystem services in the South Platte river basin.

Finally, it is also important to consider how the tradeoffs between each of the wetland functions and the potential value of agricultural production change along the output frontier. To better understand the change in shadow prices along the frontier, as the relative proportions of wetland function and agricultural value (i.e., the output shares) change, the Morishima elasticity of substitution (Blackorby and Russel, 1989), or in an output context, transformation, is also estimated for each of the wetland functions. This provides a measure of curvature for the frontier, and can be generalized to the case of multiple (more than two) outputs. As opposed to pairwise Hicks elasticities (Allen and Hicks, 1934), which hold all other outputs constant, the Morishima elsasticity allows all output prices to simultaneously adjust to a change in the output ratio, offering information on the optimal shadow price ratios. Blackorby and Russel (1989) note that the Hicks elasticity can only be used to determine the optimal output price ratio (in their case, input shares) for the case of separable outputs (inputs), which would imply an additive production function. The quadratic form of the directional output distance function specified in this study violates this separability condition. The Morishima elasticity of transformation is defined as

$$M_{mm'} = \frac{\partial ln(\frac{p_m}{p_{m'}})}{\partial ln(\frac{y_{m'}}{y_m})}. \tag{33}$$

Given the shadow price ratio for two outputs, $(y_m, y_{m'})$,

$$\frac{p_m}{p_{m'}} = \frac{\partial \vec{D_O}(x, y; g_y)/\partial y_m}{\partial \vec{D_O}(x, y; g_y)/\partial y_{m'}}, \tag{34}$$

and using the directional output distance function, the Morishima elasticity can be estimated as

$$M_{mm'} = y^*_{m'} \left[\frac{\partial^2 \vec{D_O}(x, y; g_y)/\partial y_m \partial y_{m'}}{\partial \vec{D_O}(x, y; g_y)/\partial y_m} - \frac{\partial^2 \vec{D_O}(x, y; g_y)/\partial y_{m'} \partial y_{m'}}{\partial \vec{D_O}(x, y; g_y)/\partial y_{m'}} \right], \tag{35}$$

where $y^* = y + \vec{D_O}(x, y; g_y)$. With the quadratic specification of the directional output distance function, and the empirical application, this simplifies to

$$M_{mm'} = y^*_{m'} \left[\frac{\beta_{mm'}}{\beta_m + \sum\limits_{m'=1}^{M} \beta_{mm'} y_{m'}} - \frac{\beta_{m'm'}}{\beta_{m'} + \sum\limits_{m=1}^{M} \beta_{mm'} y_m} \right]. \tag{36}$$

The properties of the directional output distance function, combined with the specified quadratic form, determine the sign of the Morishima elasticity (Färe et al., 2005). First, because the directional output distance function is concave in y,

$$\frac{\partial^2 \vec{D_O}(x, y; g_y)}{\partial y_m \partial y_m} \leqq 0, \; m = 1, ..., M,$$

the parameter value $\beta_{mm} \leqq 0$, $m = 1, ..., M$. In this application, the bias-corrected value for β_{55} is actually equal to zero, removing that term from the Morishima elasticity computation. Secondly, because the directional output distance is monotonic decreasing in y, (i.e. higher functions scores reduce distance to the frontier)

$$\frac{\partial \vec{D}_O(x, y; g_y)}{\partial y_m} \leqq 0, \; m = 1, ..., M.$$

Together, concavity and monotonicity of the directional output distance function imply that for the quadratic form, the Morishima elasticity, $M_{mm'}$ will take the opposite sign of $\beta_{mm'}$. The translation property lends additional structure to the estimation of $\beta_{mm'}$, namely that

$$\sum_{m'=1}^{M} \beta_{mm'} g_{y_{m'}} = 0, \; m = 1, ..., M.$$

The unit direction vector then implies that

$$\beta_{mm'} = -\sum_{m''=1}^{M} \beta_{mm''}, \; m'' = 1, ..., M, m'' \neq m',$$

which makes the sign of $ME_{mm'}$ ambiguous. For a negative $ME_{mm'}$, as the magnitude, or absolute value of the elasticity increases, it becomes more costly to increase y_m, while for a positive $ME_{mm'}$, greater elasticity implies that it is less costly to increase y_m. Regardless of sign, the effect of a change in the output shares on the shadow price ratio increases as $ME_{mm'}$ becomes more elastic. Table 6 lists the resulting Morishima elasticity values, constructed with the bias-corrected parameter and marginal performance estimates.

Table 7: Bias-Corrected Morishima Elasticity Estimates

Variable	Mean	Std. Dev.	Min	Max
M_{15}	-0.520	2.992	-26.991	-0.035
M_{25}	0.066	0.066	0.022	0.573
M_{35}	-0.362	0.473	-2.488	-0.091
M_{45}	-0.017	0.011	-0.091	-0.007

In this application, the average elasticities are generally negative, and inelastic, although some of the individual elasticities are highly elastic. This indicates that the shadow price ratios may be somewhat unresponsive to changes in the relative function and agricultural values. While typically quite low in magnitude, the vegetation function elasticity is positive (the shadow price ratio is still positive), which implies that as the share of agricultural value increases, it may become (slightly) less costly to improve the vegetation function.

6 Conclusion

This work extends the production frontier approach to valuation to the wetlands literature, by using the directional output distance function to derive shadow price estimates for a set of wetland functions in the Nanticoke River watershed. The estimation procedure adapts the bootstrap methods developed for nonparametric estimation to the quadratic directional output distance function. The results suggest that the value of improved wetland function outweighs the potential value of agricultural production for many of the areas within the watershed. The average estimated value of improved wetland function is also roughly in line with the permanent easement payments currently being offered by the WRP for the study area. In practice, this approach could be used to target conservation and restoration funding, by programs such as the WRP, to the wetland areas where improved condition is most valuable.

There may be advantages to using the production frontier approach, as opposed to other methods, to value wetlands in some circumstances. For instance, when trying to value wetland function, explanation of the relevant hydrological and biological processes may be overly complicated for survey-based valuation methods. Alternatively, connecting these functions to their associated ecosystem services, which may be easier to describe for contingent valuation, requires quantification of the function-to-service link. The hedonic property price methods used to value urban wetlands are not always easily transfered to rural areas, where observations are more dispersed. While it may be possible to value some wetland functions, such as groundwater recharge, as productive inputs, the input role of other functions, such as vegetation coverage, may be less apparent.

That said, there are also limitations to the production frontier approach, and more importantly, its application in this study. The output frontier is used to value the tradeoffs between wetland function and agricultural production. This measure of wetlands value reflects the opportunity cost of foregone agricultural production, a private market value, and thus is likely to underestimate the social value of wetlands. This limitation is not of course unique to the production frontier approach, and can also downward-bias hedonic and input value estimates. The assumptions made to characterize the output set may not always be realistic. For instance, if different aspects of wetland condition are closely related (in this application, the function scores have relatively low correlation statistics) the free disposability assumption may be inappropriate. Land serves as the only input in this application, which ignores other possible inputs, such as agricultural inputs or conservation measures. This application could also be enhanced by a physical model of the effects of agricultural practices (e.g., irrigation, tillage, chemical use) on each of the wetland functions.

More generally, this work illustrates a way to link the data used for ecological assessment of wetlands to economic valuation. Ecological assessment focuses largely on indicators of wetland function (such as the HGM indicators included here), yet relatively few studies in the existing wetlands valuation literature examine the value of wetland function, particularly with similar ecological indicator data. Integrating the findings from both disciplines into socially beneficial wetlands management policy requires bridging this disconnect.

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