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Abstract

The aim of this article is to present new models of acreage choices to describe short term production choices. Their construction combines concepts developed in the Positive Mathematical Programming and Multi-crop Econometric literatures. They consider land as an allocable fixed input and motivate crop diversification by decreasing returns to crop area and/or implicit costs generated by constraints on acreage choices and by limiting quantities of quasi-fixed factors. Attractive re-parametrization of the standard quadratic production function and different functional forms for cost function are proposed to have parameters easily interpretable and to define econometric models in a very simple way.

Keywords: acreage share, production function, multi-crop econometric model, positive mathematical programming

JEL classifications: D22, C51, Q12

Modélisation économétrique des choix de production de court terme des agriculteurs dans les systèmes de culture.

Résumé

L'objectif de cet article est de présenter de nouveaux modèles de choix d'assolements qui permettent de décrire les décisions de production de court terme des agriculteurs. Ces modèles s'appuient à la fois sur des concepts empruntés aux modèles de Programmation Mathématique Positive (PMP) et aux modèles économétriques multi-produits (ME). La terre est considérée comme un input fixe mais allouable et on suppose que ce sont les rendements de la terre décroissants et/ou l'existence de coûts implicites de gestion d'un assolement qui incitent les agriculteurs à diversifier leur assolement. Ces coûts implicites, utilisés dans les modèles de PMP, sont considérés être générés par des contraintes liées aux quantités limitées d'intrants quasi-fixes (le capital et le travail). Une re-paramétrisation de la forme quadratique standard est également proposée pour les fonctions de production ainsi que différentes formes fonctionnelles pour la fonction de coût. Ces modèles ont l'avantage d'avoir des paramètres facilement interprétables et permettent de définir des modèles économétriques simples.

Mots-clés : choix d'assolements, fonction de production, modèle économétrique multi-produits, programmation mathématique positive

Classifications JEL : D22, C51, Q12

Simple econometric models for short term production choices in cropping systems

1. Introduction

The aim of the paper is to develop new econometric models of acreage choices from microeconomic theory. Two main approaches are used to model crop decisions, the Mathematical Programming (MP) and the Econometric Modelling (ME). The MP has long been used to model farmer's behaviour and simulate policy changes because of its simplicity of implementation and interpretation. It has also generated numerous applications (e.g., Paris and Howitt, 1998; Paris, 2001) but also criticisms (Britz *et al.*, 2003; Heckelei and Wolff, 2003), which have motivated alternative methods. The Positive Mathematical Programming (PMP), developed by Howitt (1995), is a method to calibrate mathematical programming models. Its advantages are the simplicity of modelling technological, environmental and bio-economic constraints, the smoothness of the model responses to policy changes and a simple interpretation by policymakers. The main difference with econometrics is that PMP does not require a series of observations to reveal the economic behaviour, which deprives PMP from inference and validation tests (De Frahan, 2005). The ME also provides a useful framework to analyse policy instruments (e.g., Guyomard *et al.*, 1996; Oude Lansink and Peerlings, 1996; Moro and Sckokai, 1999). Dual models are often used to model farmers' behaviour with explicit allocation of fixed factors (Chambers and Just, 1989). Their main limitations are that additional constraints are hardly integrated and parameters of these econometric models are difficult to interpret. In short, the MP is generally used when economists need to model complex technological or political constraints or if few data are available.

In this paper, the construction of models combines the concepts developed in the PMP and ME literatures. This work is built on Heckelei and Wolff's (2003) methodological framework which aims at moving the two methodological approaches closer together. Our models rely on simple optimization concepts as well as on a primal representation of the crop production process. They consider land as an allocable fixed input and motivate crop diversification by implicit costs and/or decreasing returns to crop area. The acreage implicit cost function concept is specific to the PMP framework (Howitt, 1995) whereas decreasing returns to land is a crop diversification motive considered in the PMP as well as in the ME framework (Just *et al.*, 1983; Chambers and Just, 1989). The proposed models may be used for estimating the parameters of PMP models or as simple ME models. As a result they contribute to fill the gap between these methodological approaches. These models support several choices of

functional forms. We propose an attractive re-parametrization of the standard quadratic production function and several different functional forms for the cost function. These proposed functional forms have two main advantages. Their parameters are easy to interpret and they allow to define econometric models in a very simple way. They can thus be used in research programs or applied studies involving economists and non-economists. Our models' parameters are estimated by using usually available micro-level data as well as standard statistical tools. They share this feature with models developed in the ME literature (Chambers and Just, 1989; Moore and Negri, 1992; Oude Lansink and Peerlings, 1996, among others).

This article is organized in six main sections. Section 1 compares our modelling framework with previous work. Sections 2 and 3 propose functional forms for yield and cost functions. Section 4 provides the econometric specification of the models. Section 5 presents an empirical illustration for French crop producers aiming at comparing the performances of the proposed production models. Proofs and detailed computations are provided in appendices. The last section provides concluding remarks and proposes directions for further research.

2. Modelling framework

We denote s_k as the acreage shares of the crop k with $k = 1, \dots, K$ and $\sum_{k=1}^K s_k = 1$. Crop k output is sold at price P_k . The vector of variable input prices is denoted by \mathbf{w} . The vector \mathbf{z} describes the farmer's endowment in quasi-fixed factors while \mathbf{v} is the vector of prices of non directly productive (in an agronomical sense) variable inputs such as energy and capital maintenance inputs. The term π_k is the gross margin of the crop k .

2.1. A brief literature review on production econometric models

Quantitative models of acreage choices typically belong to one of two main methodological types: either (P)MP models or ME models consisting of dual systems of supply and input demand equations (Heckelei and Wolff, 2003). They also differ by their focus on one or two of the main motives for crop diversification: decreasing marginal return to crop acreages (or more generally scale and scope economies. See, *e.g.*, Lynne, 1988 or Leathers, 1991), (production or/and price) risk spreading, constraints associated to allocated quasi-fixed factors (other than land) or crop rotation effects. ME models considering land as fixed but allocable

mostly focus on decreasing marginal returns to crop acreage (see, *e.g.*, Just *et al.*, 1983; Chambers and Just, 1989; Moore and Negri, 1992) and on risk spreading (see, *e.g.*, Chavas and Holt, 1990; Coyle, 1992) as the motives for crop diversification. Crop rotation effects are more rarely considered in multi-crop econometric models, probably due to the complexity of dynamic choice modelling (see, *e.g.*, Eckstein, 1984; Tegene *et al.*, 1988; Ozarem and Miranowski, 1994). Recent studies tend to show a renewed interest in this topic (see, *e.g.*, Thomas, 2003; Hennessy, 2006; Livingston *et al.*, 2008). In what follows, we describe only commonly used static ME models considering land as an allocated fixed input.

The commonly used static econometric models considering land as an allocated fixed input consider two types of indirect restricted profit functions Π , either:

$$\Pi(\mathbf{s}; \mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{v}) = \sum_{k=1}^K s_k \pi_k(p_k, \mathbf{w}; \mathbf{s}, \mathbf{z}, \mathbf{v}) + H(\mathbf{s}, \mathbf{z}), \quad (1a)$$

see Chambers and Just (1989) and Oude Lansink and Peerlings (1996), among others, or:

$$\Pi(\mathbf{s}; \mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{v}) = \sum_{k=1}^K s_k \pi_k(p_k, \mathbf{w}; \mathbf{s}_{-k}; \mathbf{z}, \mathbf{v}), \quad (1b)$$

see, *e.g.*, Chambers and Just (1989), Moore and Negri (1992), Guyomard *et al.* (1996) or Moro and Sckokai (1999). In both cases, the crop specific gross margins π_k depend on (\mathbf{z}, \mathbf{v}) , implying that farmers may adapt their input uses to the available quantities of labour and/or machinery. The same remark applies for the effect of \mathbf{s}_{-k} in $\pi_k(p_k, \mathbf{w}; \mathbf{s}; \mathbf{z}, \mathbf{v})$, where \mathbf{s}_{-k} is the vector obtained by deleting s_k from \mathbf{s} . The \mathbf{s}_{-k} term cannot represent “structural” crop rotation effects because these effects characterize the dynamics of the multi-crop production process.¹ The $H(\mathbf{s}, \mathbf{z})$ term is the main distinctive feature of the restricted indirect profit functions described by equations (1a) and (1b). It is a reduced form function capturing the

¹ According to the usual structural interpretation of the multioutput technology, this framework also imposes non-jointness restrictions of the multioutput technology in variable inputs, in outputs and in acreages. Non-jointness in variable inputs and in outputs is commonly assumed while non-jointness in acreages is more debated (see, *e.g.*, Just *et al.*, 1983 ; Chambers and Just, 1989 ; Leathers, 1991 ; Asunka and Shumway, 1996).

effects of the elements of (\mathbf{s}, \mathbf{z}) “outside” the crop gross margin functions which may affect the farmer’s acreage choice.

2.2. The proposed econometric model

In our multi-output model, the farmers’ restricted profit function Π is defined as:

$$\Pi(\mathbf{s}; \mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{v}) = \sum_{k=1}^K s_k \pi_k(p_k, \mathbf{w}; s_k) - C(\mathbf{s}; \mathbf{z}, \mathbf{v}) \quad (2)$$

This model can easily accommodate the effects of (\mathbf{z}, \mathbf{v}) . Our argument for not considering the effects of $(\mathbf{z}, \mathbf{v}, \mathbf{s}_{-k})$ in the crop gross margins is twofold. First, this assumption choice is well suited if it is more profitable for farmers to adapt their land allocation choices to their available quasi-fixed input quantities rather than to adapt their variable input uses at the crop level. This is usually asserted by the agricultural scientists and the extension agents consulted by the authors. The basic idea here is that the cropping practices employed by the farmers have well-known properties within rather narrow input use levels. As a result farmers perceive “constrained” input uses as risky, at least riskier than acreage changes fitting their rotation scheme and their quasi-fixed input capacities. If this assumption holds, *i.e.* if the farmers’ input choices are truly rigid at the crop level, the estimated effects of $(\mathbf{z}, \mathbf{v}, \mathbf{s}_{-k})$ in the production functions do not reflect the “structural” effects implied by the underlying optimization model. The second argument is that, as far as short term production choices are concerned, the independence assumption of $(\mathbf{z}, \mathbf{v}, \mathbf{s}_{-k})$ with respect to variable input uses holds “locally” (and approximately), *i.e.* for variable input uses in the neighbourhood of the current use levels. In micro-econometric studies, the estimated effects of $(\mathbf{z}, \mathbf{v}, \mathbf{s}_{-k})$ may just capture, *e.g.*, part of the heterogeneity in the production conditions.

The $C(\mathbf{s}, \mathbf{z}, \mathbf{v})$ term has a functional role similar to that of the $H(\mathbf{s}, \mathbf{z})$ term, except that this function depends on \mathbf{V} due to our dichotomy of the variable inputs. In this respect, the present framework just provides an interpretation of $H(\mathbf{s}, \mathbf{z})$. The acreage implicit cost function $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$ can be interpreted as a reduced form function smoothly approximating i) the unobserved variable costs associated with a given acreage (energy costs, etc.) and ii) the effects of binding constraints on acreage choices, *e.g.* agronomic constraints or constraints

associated to limiting quantities of quasi-fixed inputs. Quasi-fixed inputs such as labour and machinery are limiting in the sense that their cost per unit of land devoted to a given crop is likely to increase due to work peak loads or due to machinery overuse, whether machinery is specific or not. Some crop rotations are impossible due to inconsistencies in planting and harvesting dates. Cultivating a given crop two consecutive years on the same plot may also be strongly unwarranted due to dramatic expected pest damages. These crop rotations are thus almost “forbidden” because their opportunity cost is very large in standard price ranges. These impossible and “forbidden” crop rotations determine the bounds imposed to acreage choices in (P)MP models. The implicit cost function $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$ is assumed to be non-decreasing and quasi-convex in \mathbf{s} to reflect the constraints due to the limiting quantities of quasi-fixed factors (other than land) and due to the implicit bounds imposed on the acreage choices due to impossible or “forbidden” crop rotations. Its definition implies that $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$ can also be assumed to be decreasing in \mathbf{z} , the available quantities of quasi-fixed inputs (other than land), as well as to be increasing in \mathbf{v} .

Implicit cost functions, similar to the one defined in (3), are used in the PMP literature (Howitt, 1995; Paris and Howitt, 1998). They are also considered in dynamic models to account for adjustment costs, see *e.g.* Oude Lansink and Stefanou (2001). Heckelevi and Wolff (2003) also propose to use this form of restricted profit function to define multi-crop econometric models with land as an allocable fixed factor. The main differences between the cost function used here or in Heckelevi and Wolff (2003) and the ones used in the PMP literature are that i) $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$ includes the effects of all binding constraints on acreage choices and ii) $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$ excludes the allocated productive (in an agronomical sense) input costs which are parts of the gross margins $\pi_k(p_k, \mathbf{w}, s_k)$ for $k = 1, \dots, K$. Arnberg and Hansen (2007) use an implicit acreage cost function on the same basis, with a strong focus on stable crop rotation schemes and work peak loads.

2.3. Optimization in two steps

We follow the approach developed by Chambers and Just (1989) and decompose the farmer’s problem into two steps. In the first step, the farmer chooses the optimal objective yield and input uses for each crop. Farmers’ per hectare gross margin functions are then defined by:

$$\pi_k(p_k, \mathbf{w}; s_k) \equiv \text{Max}_{y_k, \mathbf{x}_k} [p_k y_k - \mathbf{x}_k' \mathbf{w}] \quad \text{s.t.} \quad y_k = f_k(\mathbf{x}_k; s_k), y_k \geq 0, \mathbf{x}_k \geq \mathbf{0} \quad (3)$$

for $k = 1, \dots, K$, where y_k is the yield of crop k , \mathbf{x}_k is the quantity vector of variable input uses per unit of land of crop k and $f_k(\cdot)$ is the yield function of the crop k and is assumed to be non-decreasing and concave in \mathbf{x}_k .

In the next sections $\pi_k(p_k, \mathbf{w}; s_k)$ will denote the expected gross margin of crop k , the expectation being that of the farmer at the time he chooses his acreage. The \mathbf{x}_k vectors include inputs that are directly involved in the crop growth and development processes (as fertilizers, pesticides, seeds and energy). The other quasi-fixed inputs (mainly machinery and labour) are used for the variable input applications, for harvesting or for ploughing. Also the availability of quasi-fixed inputs mostly plays an indirect role in the biological crop production process (for a given soil preparation). The resulting yield functions only depend on variable inputs with a “direct” agronomical role and, as a result, mostly represent the biological crop production process. The role of s_k in $f_k(\mathbf{x}_k; s_k)$ is further discussed below. The main benefit of this framework is that the yield functions $f_k(\cdot)$ are similar to the ones considered by agricultural scientists. The $f_k(\cdot)$ functions are to be interpreted as response functions of the crop to the use of input quantities \mathbf{x}_k provided that some cropping practice is employed within a given long term production strategy.

The presented modelling framework main assumptions were defined following our discussions with agricultural scientists and extension agents. These “field” experts generally assert that farmers are more reluctant to change their cropping practices than their land allocation, at least in the short run and within standard rotation patterns. In the short run, farmers only moderately adapt their variable input uses as well as their yield objectives. More drastic changes in cropping patterns usually require quasi-fixed input changes as well as new cropping practices adoption, *i.e.* investments in technological changes. Furthermore, farmers usually follow stable “rotation schemes” for choosing which crop to plant on their plots. These rotation schemes define farmers’ rules of thumb which can be interpreted as guides for optimal inter-temporal acreage choices. (P)MP models constraints on acreage choices are typically drawn on these bounds. These rules of thumb mainly depend on the farm’s natural

capital endowment (*e.g.*, topography, plot dispersion or soil quality heterogeneity in the farm’s plots) and on the past, current and anticipated future economic context.

The models presented in this article consider short term choices within a given multi-crop production technology as it is defined by the crop rotation scheme and the cropping practices employed by the considered farmer. Moderate changes in cropping practices, *e.g.* moderate changes in fertilizer use or pesticide use levels, do not change the short run production technologies, *i.e.* the $f_k(\cdot)$ functions, and only slightly modify the requirements for the quasi-fixed input services. Drastic changes in cropping practices involve long term choices: adoption of new cropping practices and/or adoption of new rotation schemes involving changes in the yield functions $f_k(\cdot)$ and adaptation of the quasi-fixed input quantities.²

In the second step, the farmers’ restricted profit function explicitly defines a trade-off between the crop gross margins $\pi_k(p_k, \mathbf{w}, s_k)$ of the different crops on the one hand and the “implicit management cost” of the chosen allocation $C(\mathbf{s}, \mathbf{z}, \mathbf{v})$ on the other hand:

$$\Pi(\mathbf{s}; \mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{v}) = \sum_{k=1}^K s_k \pi_k(p_k, \mathbf{w}; s_k) - C(\mathbf{s}; \mathbf{z}, \mathbf{v}) \quad (4)$$

This restricted indirect profit function embeds two motives for crop diversification: the effects of the s_k ’s on the return of crop k , and the cost function $C(\mathbf{s}; \mathbf{z}, \mathbf{v})$.

The proposed model aims at describing short term production choices and is essentially static. This feature is implicitly shared by many of commonly used acreage choices models. It can be interpreted as a local approximation of the true choice process of the farmers, *i.e.* it is only valid in a neighbourhood of the farmers’ current short run choices.

² Changes in rotation schemes may be induced by drastic changes in the anticipated future economic context. For example, a drastic increase in pesticide prices may provide strong economic incentives for adopting longer rotation schemes for benefiting from the beneficial effects of crop rotations with respect to pests and diseases control. This change in the economic context would also change the cropping practices used by farmers, for two reasons. First, it may appear more profitable for farmers to reduce their yield objectives according to a standard relative price effect. Second, adoption of a new rotation scheme reduces the need for chemical control of pests and diseases according to a “technological” change effect.

3. The “translated” quadratic yield function

The “translated” quadratic functional form is chosen for the yield functions for three reasons. First its congruent dual functions have simple functional forms. Second, the quadratic production function can be parameterized in a form which is fairly easy to interpret by agricultural scientists or extension agents. Third, the resulting yield supply, input demand and (indirect) gross margin functions can be generalized to account for farms and farmers unobserved heterogeneity and for production stochastic events in a “natural” way, *i.e.* by introducing additive random terms with simple interpretations. Pope and Just (2003) or Bandiera and Rasul (2006) used this parameterization of the quadratic production function for this last reason, albeit in different contexts.

The “translated” quadratic yield functions are defined as:

$$y_k = a_k - 1/2 \times (\mathbf{b}_k - \mathbf{x}_k)' \Gamma_k^{-1} (\mathbf{b}_k - \mathbf{x}_k) \quad (5)$$

for $k = 1, \dots, K$. This function is a simple re-parameterisation of the “standard” quadratic yield functions, $y_k = a_k^s + \mathbf{x}_k' \mathbf{b}_k^s - 1/2 \times \mathbf{x}_k' \Gamma_k^{-1} \mathbf{x}_k$ with $\mathbf{b}_k \equiv \Gamma_k \mathbf{b}_k^s$ and $a_k \equiv a_k^s + 1/2 \times \mathbf{b}_k' \Gamma_k^{-1} \mathbf{b}_k$. In this primal framework, the a_k and \mathbf{b}_k terms have direct interpretations: \mathbf{b}_k is the variable productive input quantity required to achieve the maximum yield a_k . Both terms need to be positive. The Γ_k^{-1} matrix determines the curvature of the yield function and, as a result, determines the magnitude of the price effects. It needs to be positive definite for the yield function to be strictly concave. This also implies that $y_k \leq a_k$ for any input choice vector \mathbf{x}_k . Estimates of these parameters can easily be “checked” with agricultural scientists and extension agents.

This yield function can easily be accommodated for empirical purposes by specifying a_k and \mathbf{b}_k as functions of observed factors affecting crop k production process.³ The maximum yield and input requirement parameters, a_k and \mathbf{b}_k , can be defined as functions of s_k accounting

³ The specification of the curvature matrix as a function of observed variables is likely to generate empirical difficulties.

for the marginal effects of acreage level of crop k . Assuming that the effect of s_k in a_k and \mathbf{b}_k is linear, *i.e.* that $a_k = \alpha_k + \alpha_{s,k} s_k$ and $\mathbf{b}_k = \boldsymbol{\beta}_k + \boldsymbol{\beta}_{s,k} s_k$, decreasing marginal returns are implied by $\alpha_{s,k} \leq 0$ and by $\boldsymbol{\beta}_{s,k} \geq \mathbf{0}$. More importantly, a_k and \mathbf{b}_k can easily be adapted to account for heterogeneity of the production conditions across farms and years. From the econometrician viewpoint, the terms α_k and $\boldsymbol{\beta}_k$ are random parameters. But these terms can also be considered as partially random from the farmer's viewpoint.

Let now decompose a_k and \mathbf{b}_k as $a_k = \alpha_{0,k} + \alpha_{s,k} s_k + e_k^\alpha + \varepsilon_k^\alpha$ and $\mathbf{b}_k = \boldsymbol{\beta}_{0,k} + \boldsymbol{\beta}_{s,k} s_k + \mathbf{e}_k^\beta + \boldsymbol{\varepsilon}_k^\beta$ where $\alpha_{0,k}$ and $\boldsymbol{\beta}_{0,k}$ are the means of α_k and $\boldsymbol{\beta}_k$ across farms and time (provided that s_k is fixed), and $\alpha_{0,k} + \alpha_{s,k} s_k + e_k^\alpha$ and $\boldsymbol{\beta}_{0,k} + \boldsymbol{\beta}_{s,k} s_k + \mathbf{e}_k^\beta$ are the expectations of a_k and \mathbf{b}_k (provided that s_k is fixed) from the farmer's point of view at the beginning of the cropping campaign, *i.e.* at the time acreage is decided. In that case, ε_k^α and $\boldsymbol{\varepsilon}_k^\beta$ define the effects of the stochastic events affecting crop k production process during the cropping campaign, *e.g.* the effects of climatic events or pest and disease damages.⁴ We assume that the farmer observes $\boldsymbol{\varepsilon}_k^\beta$ during the production process⁵ and that he can use this information for updating his variable input choices. It is also assumed that the input and product k prices are known at the beginning of the growing season. In the case where p_k is only known at the harvest time, p_k and \mathbf{w}_k need to be replaced by their expectations (from the farmer's viewpoint).

Standard expected profit maximisation arguments allow to show that the farmer's variable input demand and supply functions are given by (see Appendix):

$$\mathbf{x}_k(\mathbf{w}_k; s_k) + \boldsymbol{\varepsilon}_k^\beta = \boldsymbol{\beta}_{0,k} + \boldsymbol{\beta}_{s,k} s_k + \mathbf{e}_k^\beta - \boldsymbol{\Gamma}_k \mathbf{w}_k + \boldsymbol{\varepsilon}_k^\beta \quad (6a)$$

and:

$$y_k(\mathbf{w}_k; s_k) + \varepsilon_k^\alpha = \alpha_{0,k} + \alpha_{s,k} s_k + e_k^\alpha - 1/2 \times \mathbf{w}_k' \boldsymbol{\Gamma}_k \mathbf{w}_k + \varepsilon_k^\alpha \quad (6b)$$

⁴ This “purely” structural interpretation ignores measurement or optimisation errors (see, *e.g.*, Pope and Just, 2003)

⁵ Whether the farmer observes ε_k^α or not does not change his optimal input choices. The effects of ε_k^α (as well as those of e_k^α) are necessarily entirely foregone by the farmer.

where $\mathbf{w}_k \equiv \mathbf{w}/p_k$ for $k = 1, \dots, K$.⁶ These solutions are the estimable yield supply and input demand functions. At the beginning of the cropping campaign, the farmer's expected gross margins for crop k is given by $\pi_k(p_k, \mathbf{w}; s_k) \equiv p_k y_k(\mathbf{w}_k; s_k) - \mathbf{w}' \mathbf{x}_k(\mathbf{w}_k; s_k)$ and thus by:

$$\pi_k(p_k, \mathbf{w}; s_k) = p_k(\alpha_{0,k} + \alpha_{s,k} s_k + e_k^\alpha) - \mathbf{w}'(\boldsymbol{\beta}_{0,k} + \boldsymbol{\beta}_{s,k} s_k + \mathbf{e}_k^\beta) + 1/2 \times \mathbf{w}' \boldsymbol{\Gamma}_k \mathbf{w}_k \quad (6c)$$

for $k = 1, \dots, K$. Ignoring the effects of ε_k^α and ε_k^β in the definition of the expected gross margins is important. In the case where input use levels \mathbf{x}_k are observed together with the obtained yield y_k , it is possible to compute the obtained gross margin levels, *i.e.* $\pi_k \equiv p_k y_k - \mathbf{w}' \mathbf{x}_k$. But the risk-neutral farmer chooses his acreage according to the $\pi_k(p_k, \mathbf{w}_k; s_k)$'s since he does not know the π_k 's. Equations (6a)-(6c) provide the basis of the yield and input use equations of all our ME models.

4. Simple acreage choice models

This section presents three models of acreage choices. Specifically, we consider different functional forms for⁷:

$$\Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})] = \mathbf{s}' \boldsymbol{\pi}(\mathbf{s}) - C(\mathbf{s}), \quad (7a)$$

each of them leading to closed-form solutions to the maximisation problem:

⁶ This framework also accommodates cases where some of the variable inputs cannot be adjusted during the growing season (see Appendix).

⁷ In order to simplify the notations, we note $\boldsymbol{\pi}(\mathbf{s}) \equiv (\pi_1(p_1, \mathbf{w}; s_1), \dots, \pi_K(p_K, \mathbf{w}; s_K))$, we omit the (\mathbf{z}, \mathbf{v}) argument in $C(\mathbf{s})$ and note $\Pi(\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})) \equiv \Pi(\mathbf{s}; \mathbf{p}, \mathbf{w}, \mathbf{z}, \mathbf{v})$.

$$\text{Max}_s \Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})] \quad \text{s.t.} \quad \mathbf{s} \geq \mathbf{0} \quad \text{and} \quad \mathbf{s}'\mathbf{j} \leq 1 \quad (7b)$$

where \mathbf{j} is the unitary vector of dimension K . Three of the presented models rely on the constant returns to crop acreage assumption for each crop, *i.e.* $\Pi(\mathbf{s}; \boldsymbol{\pi}) = \mathbf{s}'\boldsymbol{\pi} - C(\mathbf{s})$. One of the models considers that the implicit acreage costs can be neglected, *i.e.* $\Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})] = \mathbf{s}'\boldsymbol{\pi}(\mathbf{s})$. And finally one of the models considers returns to crop acreage and implicit acreage costs, *i.e.* $\Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})] = \mathbf{s}'\boldsymbol{\pi}(\mathbf{s}) - C(\mathbf{s})$. If the available data contain the crop gross margins, the acreage share models assuming constant returns to crop acreage can directly be used.

4.1. Quadratic cost acreage model with decreasing returns to crop acreage (QCDR model)

PMP models usually rely on quadratic implicit cost functions (Howitt, 1995; Heckelei and Wolff, 2003). In the general case, the considered indirect restricted profit function has the following form:

$$\Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})] = \sum_{k=1}^K s_k \pi_k - \sum_{k=1}^K \delta_k s_k^2 - \left[A + \sum_{k=1}^K c_k s_k + 1/2 \times \sum_{k=1}^K \sum_{m=1}^K c_{km} s_k s_m \right] \quad (8a)$$

where $\delta_k \equiv (p_k \alpha_{s,k} - \mathbf{w}'\boldsymbol{\beta}_{s,k})$ and $C(\mathbf{s}) \equiv A + \sum_{k=1}^K c_k s_k + 1/2 \times \sum_{k=1}^K \sum_{m=1}^K c_{km} s_k s_m$ is the acreage implicit cost function, A is an unidentifiable fixed cost and $c_{km} = c_{mk}$ for $k, m = 1, \dots, K$. If the matrix $\mathbf{C} \equiv [c_{km}, k, m = 1, \dots, K]$ is definite positive, the cost function $C(\mathbf{s})$ is strictly convex in \mathbf{s} . Assuming that the solution, denoted by $\mathbf{s}(\boldsymbol{\pi}, \boldsymbol{\delta})$ with $\boldsymbol{\delta} \equiv (\delta_1, \dots, \delta_K)$, to the maximisation problem (7b) is unique and interior, it is characterized by the equation system:

$$\begin{aligned} (\pi_k - \pi_K) - (c_k - c_K) - (c_{kK} - c_{KK}) + 2 \times \delta_K \\ - 2 \times \delta_k s_k(\boldsymbol{\pi}, \boldsymbol{\delta}) - \sum_{m=1}^{K-1} ((c_{km} - c_{Km}) + (c_{kK} - c_{KK}) + 2 \times \delta_K) s_m(\boldsymbol{\pi}, \boldsymbol{\delta}) = 0 \end{aligned} \quad (8b)$$

for $k = 1, \dots, K - 1$ and:

$$s_K(\boldsymbol{\pi}, \boldsymbol{\delta}) = 1 - \sum_{k=1}^{K-1} s_k(\boldsymbol{\pi}, \boldsymbol{\delta}). \quad (8c)$$

The land use binding constraint leading to equation (8c) is used to determine equation (8b). Equations (8b) and (8c) are simply derived from the first order conditions of problem (7b) as applied with the functional form of $\Pi[\mathbf{s}; \boldsymbol{\pi}(\mathbf{s})]$ given by equation (8a). Once again crop K serves as the reference without loss of generality. These equations can serve as a basis for estimating the $(c_k - c_K) + (c_{kK} - c_{KK})$ and $(c_{km} - c_{Km}) - (c_{kK} - c_{KK})$ terms for $k, m = 1, \dots, K - 1$. The equation system (8b)-(8c) admits a closed form solution in $\mathbf{s}(\boldsymbol{\pi}, \boldsymbol{\delta})$. Standard matrix notations allows to write the cost function as $C(\mathbf{s}) = A + \mathbf{s}'\mathbf{c} + 1/2 \times \mathbf{s}'\mathbf{C}\mathbf{s}$. Defining the diagonal matrix Δ by $\Delta \equiv 1/2 \times \text{Diag}(\boldsymbol{\delta})$, the closed-form solution for the optimal acreage choice is defined as:

$$\mathbf{s}(\boldsymbol{\pi}, \boldsymbol{\delta}) = (\mathbf{C} + \Delta)^{-1} [(\boldsymbol{\pi} - \mathbf{c}) - \mathbf{j} \times \lambda(\boldsymbol{\pi}, \boldsymbol{\delta})] \quad (8d)$$

with:

$$\lambda(\boldsymbol{\pi}, \boldsymbol{\delta}) = \frac{\mathbf{j}'(\mathbf{C} + \Delta)^{-1}(\boldsymbol{\pi} - \mathbf{c}) - 1}{\mathbf{j}'(\mathbf{C} + \Delta)^{-1}\mathbf{j}} \quad (8e)$$

being the Lagrange multiplier associated to the binding land use constraint. This closed-form solution is of little use for estimations purpose due to the Δ term. Estimating equations derived from the FOC equations (8b) are much easier to handle. The last two considered models are easily derived from the QCDR model.

4.2. Quadratic cost acreage model (QC model)

The QC model is derived from QCDR model by imposing $\mathbf{\Lambda} = \mathbf{0}$. This model is the basis for most PMP calibration exercise. Its use as an ME model was suggested by Heckelei and Wolff (2003).

Estimating equations for the QC acreage function parameters can be derived from the FOC equations:

$$(\pi_k - \pi_K) - (c_k - c_K) - (c_{kK} - c_{KK}) - \sum_{m=1}^{K-1} s_m(\boldsymbol{\pi}, \boldsymbol{\delta}) \times ((c_{km} - c_{Km}) - (c_{kK} - c_{KK})) = 0 \quad (9)$$

for $k = 1, \dots, K - 1$.

4.3. Decreasing Return acreage model (DR model)

The DR model is derived from QCDR model by imposing $\mathbf{C} = \mathbf{0}$. This model is the basis for most PMP calibration exercise. Similar models were proposed by Just *et al.* (1983), Chambers and Just (1989) or Moore and Negri (1992). Estimating equations for the DR acreage function parameters can be derived from the FOC equations:

$$(\pi_k - \pi_K) - (c_k - c_K) - 2 \times (\delta_k s_k(\boldsymbol{\pi}, \boldsymbol{\delta}) - \delta_K s_K(\boldsymbol{\pi}, \boldsymbol{\delta})) = 0 \quad (10)$$

for $k = 1, \dots, K - 1$.

5. Specification of the econometric models

Two types of ME can be derived from the “ingredients” described in the two preceding sections, depending on the available data. For example, the micro-level data available in the European FADN (Farm Accountancy Data Network) sometimes provide measures of the per crop variable input uses (\mathbf{x}_k) but usually do not provide this very useful information. This section mainly focuses on the specification of the ME model random terms and on identification issues. In any case, the ME considered here are defined as systems of equations containing K crop yield supply equations and $K - 1$ estimating equations for the crop acreage

choices. The random terms of the “econometric” yield supply equations are those defined in the “economic” yield supply functions. The random terms of the crop acreage estimating equations are composite, they contain the random elements of the expected gross margins as well as the random terms of our specification of the $(c_k - c_K)$ and $(c_k - c_K) + (c_{kK} - c_{KK})$ terms:

$$c_k - c_K = \lambda_{0,k} + \lambda'_{z,k} \mathbf{z} + e_k^c \quad (11a)$$

and:

$$c_k - c_K + c_{kK} - c_{KK} = \lambda_{0,k} + \lambda'_{z,k} \mathbf{z} + e_k^c \quad (11b)$$

for $k = 1, \dots, K - 1$. The specification of the $(c_k - c_K) + (c_{kK} - c_{KK})$ terms need to be defined for the QCDR and QC acreage share models while the specification of $(c_k - c_K)$ suffices with the DR acreage share models. As an illustration, we describe here the ME models built by using the QCDR acreage share choice model with a single (aggregated) variable input (implying that $\mathbf{x}_k \equiv x_k$, and in particular that $\Gamma_k \equiv \gamma_k$).

5.1 Models with data on input uses at the crop level

In the favourable case where measures of x_k are available for $k = 1, \dots, K$, the following equation system is empirically tractable:

$$\left\{ \begin{array}{l} y_{kn} = \alpha_{0,k} + \alpha_{s,k} s_{kn} - 1/2 \times \gamma_k w_{kn}^2 + e_{kn}^\alpha + \varepsilon_{kn}^\alpha \\ x_{kn} = \beta_{0,k} + \beta_{s,k} s_{kn} - \gamma_k w_{kn} + e_{kn}^\beta + \varepsilon_{kn}^\beta \\ \pi_{\ell n} - \pi_{Kn} - (\lambda_{0,\ell} + \mathbf{z}'_n \lambda_{z,\ell}) + 2 \times (p_{Kn} \alpha_{s,K} - w_n \beta_{s,K}) \\ \quad - 2 \times (p_{\ell n} \alpha_{s,\ell} - w_n \beta_{s,\ell}) \times s_{\ell n} - \sum_{m=1}^{K-1} s_{mn} \times (\theta_{\ell m} + 2 \times (p_{Kn} \alpha_{s,K} - w_n \beta_{s,K})) = e_{\ell n}^s \\ \text{with } k = 1, \dots, K \text{ and } \ell = 1, \dots, K - 1 \end{array} \right. \quad (12)$$

where $\theta_{\ell m} \equiv (c_{\ell m} - c_{K m}) + (c_{\ell K} - c_{K K})$ and $e_{\ell n}^s \equiv e_{\ell n}^c + (p_{\ell n} e_{\ell n}^\alpha - w_n e_{\ell n}^\beta) - (p_{K n} e_{K n}^\alpha - w_n e_{K n}^\beta)$ for $\ell = 1, \dots, K-1$. The n subscript denotes the n^{th} observation of the considered sample with $n = 1, \dots, N$. In cross-section data sets n denotes a farmer. In panel data or series of cross-section data sets n denotes a pair farmer/year. Identification works as follows. The yield supply and input demand equations identify the $\alpha_{0,k}$, $\alpha_{s,k}$, $\beta_{0,k}$, $\beta_{s,k}$ and γ_k parameters while the acreage share choice equations identify the $\lambda_{0,\ell}$, $\lambda_{z,\ell}$ and $\theta_{\ell m}$ parameters. Note also that the acreage share choice equation sub-system alone identifies the $\alpha_{0,k}$, $\alpha_{s,k}$, $\beta_{0,\ell} - \beta_{0,K}$, $\beta_{s,k}$ and γ_k parameters along with the $\lambda_{0,\ell}$, $\lambda_{z,\ell}$ and $\theta_{\ell m}$ parameters.

5.2. Models with data on input uses at the farm level

In the less favourable case where measures of x_k are not available, the corresponding ME models can be defined by the equation systems defined in (12), but without the input demand equations. In both cases, the yield supply sub-system identifies the $\alpha_{0,k}$ and γ_k parameters, as well as the $\alpha_{s,k}$ parameters in the QCDR ME model. The acreage share choice equations identify the $\beta_{0,k} - \beta_{0,K}$ and $\beta_{s,k}$ parameters along with the $\lambda_{0,k}$, $\lambda_{z,k}$ and θ_{km} parameters in the QCDR case. Identification of the $\beta_{0,k} - \beta_{0,K}$ and $\beta_{s,k}$ terms requires the input price to be sufficiently variable across the sample observations. The (per hectare) input uses, denoted by x_n , are generally measured at the farm level. The equation $x_n = \sum_{k=1}^K s_{kn} x_{kn}$ defining x_n suggests the use of the following “total” input use equation:

$$x_n = \sum_{k=1}^K s_{kn} \times (\beta_{0,k} + \beta_{s,k} s_{kn} - \gamma_k w_{kn}) + e_n^x \quad (13)$$

with $e_n^x \equiv \sum_{k=1}^K s_{kn} \times e_{kn}^\beta + \sum_{k=1}^K s_{kn} \times \varepsilon_{kn}^\beta$ for the identification of the $\beta_{0,K}$ and “facilitating” that of the $\beta_{s,k}$ and $\beta_{0,k} - \beta_{0,K}$ parameters. Nevertheless, if we have $E[s_{kn} \times \varepsilon_{kn}^\beta] = 0$ by construction, the terms $E[s_{kn} \times e_{kn}^\beta]$ cannot be null. The ε_{kn}^β terms are observed by farmer n after his acreage decision but he accounts for the e_{kn}^β terms when choosing his acreage. Indeed

the e_{kn}^β terms appear in the closed-form definitions of the acreage shares s_{kn} . The use of the input allocation equation (13) calls for specific solutions, *e.g.* the use of control or correction functions for the $\sum_{k=1}^K s_{kn} \times e_{kn}^\beta$ random term (see, *e.g.*, Wooldridge, 2008), which are out of the scope of this article.

6. Illustrative applications

This section presents illustrative applications of our short term production choice modelling framework. It considers this estimation of three ME models with micro-level farm accountancy data.

6.1. Data

The data consists of a sample of 4,000 observations of French grain crop producers over the years 1995 to 2006, obtained from the French FADN. It provides detailed information on crop productions and prices at the farm gate. The French FADN only provides aggregate data on variable input (pesticides, fertilizers, seeds and energy) expenditures whereas input price indices are made available at the regional level. Variable input quantities are aggregated into a single variable input for simplicity. Acreage choices of three crops are considered: wheat, other cereals (mainly barley and corn) and, oilseeds (mainly rapeseed) and protein crops (mainly peas). Root crops (sugar beets and potatoes) acreages were assumed to be exogenous due to the sugar beet quota system implemented in the UE and because most of the potato acreages are defined by contracts. Fodder crop acreage (mainly silage corn) was also considered as exogenous due to feeding constraints.

6.2. Estimation issues

Because variable input uses are not measured at the crop level in our data set, the estimated models consist in a system containing 3 yield supply equations and 2 acreage share choice equations. Crop acreages are assumed to not affect input uses, *i.e.* we assume that $\beta_{k,s} = 0$, while crop acreage effects are considered in the ME specifications using the QCDR and DR acreage choice models. The three models mainly differ with respect to the use of the

presented acreage share choice models, namely the QCDR, QC and DR acreage share choice models.

Quadratic trends accounting for disembodied technological progress, past potato and sugar-beet acreages accounting for rotation effects and irrigated area were used as control variables in the yield equations. The share of cereals except corn in the aggregate “other cereals” and the share of protein crops in the aggregate “oilseeds and protein crops” are also taken into account. Their effects were linearly included in the $\alpha_{k,0}$ parameters. Physical capital and labour variables are used as control variables in the acreage share equations of the considered ME specifications, *i.e.* define the \mathbf{z}_n vector for all ME specifications.

The random terms of the models, *i.e.* the e_{kn}^α , ε_{kn}^α , e_{kn}^β and e_{ln}^c terms, are normalized to have null expectations. The price and control variables are considered as exogenous with respect to the random terms of the ME models, according to standard independence arguments applying in dual production models.

The five model parameter sets were estimated within the Generalized Method of Moments (GMM) framework to account for the heteroskedasticity of the e_{ln}^s terms and the correlation across the e_{kn}^α , ε_{kn}^α and e_{ln}^s terms. In order to facilitate comparisons across model estimates, we only consider estimating moment conditions which do not exploit the parameter restrictions across the yield supply and acreage share choice equations. These moment conditions are defined as orthogonality conditions crossing each system equation error term with specific instrument vectors. Instruments are defined for identifying all parameters. An estimated instrument for s_{kn} denoted by $\hat{s}_{kn} \equiv \hat{s}_k(\mathbf{p}_n, w_n, \mathbf{z}_n, \mathbf{t}_n)$ can be necessary in yield supply equations.⁸ The resulting GMM estimator is robust to heteroskedasticity of unknown form and does not exclude correlation of the error terms across equations. Of course, more efficient estimators can be used by exploiting the inter-equation restrictions on the parameters implied by the models’ structures and by exploiting additional orthogonality conditions.

⁸ These (consistent) estimates of the s_{kn} variables are constructed in two steps. The models’ parameters are (consistently) estimated using a GMM estimator using 1 and the elements of $(\mathbf{p}_n, w_n, w_{1n}, \dots, w_{kn}, \mathbf{z}_n, \mathbf{t}_n, \mathbf{r}_n)$ and their square- and cross-products to define the instrument vector used for each equation of the considered system. Then, the acreage share functions are used to construct (consistent) estimates of the s_{kn} terms.

6.3. Results

Table 1 presents parameter estimates of yield supply and acreage shares equations for the QC, DR and the QCDR models. Models yield quite similar results with respect to the yield supply function parameters. The fit of the models to these micro-level data is correct and almost all parameters are significantly different from zero at least at the 10% confidence level. The R^2 criteria lie between 0.12 and 0.23 for oilseeds and protein crops, 0.17 and 0.30 for wheat and 0.16 and 0.28 for other cereals. The price variables perform reasonably well. Concavity properties of yield function are respected without imposing constraints.

[Table 1 around here]

Estimates of the maximum yield are in the ranges expected by the agricultural scientists and extension agents the authors have consulted. Wheat has a higher yield than other cereals, oilseeds and protein crops. Heterogeneity control variables have significant expected “crude” effects. Trends accounting for technological progress are significant and positive for all crops. Trends squared are significant and negative. As expected, the irrigation has positive effects on yield especially for other cereals. Corn is much irrigated in France. Past acreages of sugar beets and potatoes have a positive and important effect on cereals yield. These effects are consistent with the known beneficial effects of root crops at the beginning of the crop rotation sequence. Variables corresponding to the composition of the aggregate “others cereals” and “oilseeds and protein crops” have the expected signs. It means that corn has more important yield than other cereals and that oilseeds have a less important yield than protein crops. All effects of the control variables on maximum yield are those expected. These results demonstrate that multi-crop models provide satisfactory econometric modeling frameworks.

In the DR and QCDR models, acreage share of a crop is integrated in its yield supply function in order to account for decreasing returns to land. These estimates are significant and have expected signs. According to the agricultural economics literature yield supply function and then crop gross margins are often expected to be decreasing in crop acreage because of crop rotation effects and land quality heterogeneity within the farm plots. As the acreage allocated to a given crop increases, farmers need to allocate land with less favourable crop rotation effects or less suitable land for the considered crop. The effect of acreage share in the yield

supply function, and as a result in gross margin, can be considered as a crude approximation of the effects described above.

Estimates of the acreage equations lead to more contrasted conclusions depending on considered acreage share models. In both models concavity properties of profit function are respected without imposing constraints. The R^2 criterion is 0.11 and 0.12 for QC model, 0.13 and 0.21 for DR model and 0.17 and 0.23 for QDCR model. This is relatively correct. The parameter denoted by fixed costs is estimated in all models. We cannot compare the estimates of this parameter across all models because it has not the same interpretation from one model to another. In DR models, it reflects only the difference on fixed costs $(c_k - c_K)$ between wheat (or other cereals) with the reference crop (oilseeds and protein crops). These costs represent fixed management costs related to agronomic constraints or constraints associated to availability of machinery and labor on acreage choices. In QC and QDCR models, it includes in addition to the difference of fixed costs, some quadratic terms of the cost function $(c_k - c_K) + (c_{kk} - c_{KK})$. In all cases, this parameter depends on physical capital and labour variables. These estimated “crude” effects are quite similar between all models. The quadratic costs represent the term $(c_{lm} - c_{Km}) + (c_{lK} - c_{KK})$ for QC and QDCR models. The diversification matrix is a 2×2 matrix and corresponds to different terms according to the model. This matrix contains elements which define motives of crop diversification for farmers: scale effects and/or quadratic cost terms.

Table 2 presents price elasticities of the crop acreage shares and own-price elasticities of yield functions. The signs of the own-price elasticities are as expected for all crops. They range from 0.82 to 2.65 for QC model, from 0.29 to 3.31 for DR model and from 0.66 to 2.36 for QDCR model. All cross-price elasticities are negative. Cross price elasticities show that wheat and other cereals are substitutable. A 10% increase in wheat prices causes a decrease of over 20% of others cereals acreage share with all models. On the other side cereals are quite inelastic to the price of oilseeds and protein crops. A 10% increase in oilseeds and protein crops prices causes a maximum decrease of 3.7% of other cereals acreage share. Own-price elasticities of yield functions are quite inelastic. They range from 0.16 to 0.61. Guyomard *et al.* (1996) have also estimated such price elasticities on French data. Their results show that these elasticities are between 0.2 and 0.4.

These results globally indicate that these models provide sensible results with respect to price effects and heterogeneity control variables effects and call for improvement of the

econometric model with the respect to the use of extra variables to better control for heterogeneity.

[Table 2 around here]

7. Concluding remarks

The multi-crop econometric models built in this paper aim at describing short term production choices. They combine concepts developed in the (P)MP and ME literatures. They consider land as an allocable fixed input and motivate crop diversification by decreasing returns to crop area and/or implicit costs. These costs used in the PMP literature are generated by constraints on acreage choices and/or by limiting quantities of quasi-fixed factors. On the other side our models' parameters can be estimated by using usually available micro-level data. Applications to French data at a farm level illustrate the empirical relevance of the proposed production models and allow to compare their respective performance.

Thanks to their simple structure, these models appear to be useful tools for investigating farmers' short run production decisions. They can be used to produce simple comparative statics results. They can also be used to build simple and reliable multi-crop econometric models as shown by the illustration presented in this article. Economists involved in multi-disciplinary research projects may also find them useful for defining production choice models which are likely to be preferred to the standard multi-crop dual models by non-economists thanks to the immediate interpretation of their parameters. These models also share another advantage with Mathematical Programming models: thanks to their simple structure they can easily be used for investigating the effects of new cropping practices on land allocation. Finally, these models can also be used by researchers as simple acreage choice models in more elaborated econometric models of production choice models.

Another benefit of these models is that they are consistent in their deterministic and random parts. The random parts of the proposed models are defined for representing the effects of stochastic events affecting the agricultural production process and/or the effects of non-observed farms' heterogeneity. This feature can be useful in responding to econometric problem as the variable input allocation.

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Table 1: Estimates of the Yield and Acreage Shares Equations, 1994-2007.

Explanatory Variable	<i>QC model</i>			<i>DR model</i>			<i>QDCR model</i>		
	Wheat	Other cereals	Oilseeds protein crops	Wheat	Other cereals	Oilseeds protein crops	Wheat	Other cereals	Oilseeds protein crops
Yield supply equations									
Price effects	2.85***	1.15***	3.16***	2.16***	1.53**	3.86***	3.40***	2.59*	3.82***
Average potential yield	9.30***	8.38***	7.34***	10.53***	9.46***	9.02***	11.61***	10.45***	8.65***
Constant	8.86***	8.85***	7.31***	10.17***	9.67***	8.88***	11.18***	10.13***	9.69***
Trend	0.32***	0.21***	0.07***	0.27***	0.28***	0.08***	0.29***	0.38***	0.05***
Trend square	-0.02***	-0.01***	-0.007***	-0.02***	-0.01***	-0.008***	-0.01***	-0.01***	-0.007***
Sugar beets past acreage	1.46***	1.97***	0.79*	4.61***	2.82***	-1.87***	4.57***	3.60***	-1.39**
Potatoes past acreage	2.09***	1.09	-0.37	1.73***	0.95	-0.56	1.58***	1.30	-0.09
Irrigation	0.30	1.84***	0.44***	0.55	2.71***	0.05	0.81	3.41***	0.50**
Total area	< 0.001*	< 0.001**	< 0.001**	< 0.001***	< 0.001	0.001**	< 0.001	< -0.001	0.001**
Composition aggregate	-	-1.28***	0.79***	-	-1.42***	0.83***	-	-1.49***	-0.15
Scale effects	-	-	-	-3.29***	-3.28***	-5.17***	-4.38**	-4.75***	-3.73***
<i>R-square</i>	0.30	0.28	0.23	0.23	0.23	0.12	0.17	0.16	0.12
Acreage shares equations									
Fixed costs	-22.94***	-33.47***	-	2.48***	-2.17***	-	-0.21	-8.78***	-
Constant	-21.52***	-31.85***	-	3.12***	-1.47	-	0.35	-8.00***	-
Capital	< -0.001***	< -0.001***	-	< -0.001***	-0.001***	-	< -0.001***	< 0.0011***	-
Labor	-0.62***	-0.70***	-	-0.23***	-0.22***	-	-0.19***	-0.24*	-
Quadratic costs	-37.24***	-48.81***	-35.94***	-	-	-	-0.67	9.87***	6.77***
Diversification matrix	-37.24***	-48.81***	-35.94***	-18.47***	-18.12***	-11.30***	-18.37***	-27.92***	-14.93***
<i>R-square</i>	0.12	0.11	-	0.21	0.13	-	0.23	0.17	-

Note: *, ** and *** denote parameter estimates statistically different from zero at, respectively, the 10%, 5% and 1% confidence levels.

Table 2: Average price elasticities of acreage shares and yield functions, 1994-2007.

	Acreage shares			Yield functions
	Price of wheat	Price of other cereals	Price of oilseeds, protein crops	Own-price elasticities
			<i>QC model</i>	
Wheat	1.87 (0.78)	-1.31 (0.79)	-0.37 (0.17)	0.34 (0.13)
Other cereals	-2.78 (1.76)	2.65 (1.70)	-0.07 (0.05)	0.16 (0.06)
Oilseeds, protein crops	-0.99 (0.54)	-0.09 (0.05)	0.82 (0.48)	0.50 (0.19)
			<i>DR model</i>	
Wheat	1.77 (0.32)	-1.05 (0.61)	-0.06 (0.11)	0.26 (0.10)
Other cereals	-2.20 (1.31)	3.31 (1.09)	-0.15 (0.26)	0.21 (0.08)
Oilseeds, protein crops	-1.34 (0.72)	-1.32 (0.75)	0.29 (0.52)	0.61 (0.23)
			<i>QDCR model</i>	
Wheat	1.97 (0.79)	-1.00 (0.59)	-0.25 (0.17)	0.40 (0.15)
Other cereals	-2.09 (1.24)	2.36 (1.41)	-0.13 (0.12)	0.35 (0.14)
Oilseeds, protein crops	-1.83 (0.98)	-0.44 (0.27)	0.66 (0.56)	0.61 (0.23)

Note: Standard errors are in parentheses.

Appendix: The “translated” quadratic yield function and its congruent dual functions

In this appendix, we consider a “translated” quadratic yield of the form:

$$y = a - 1/2 \times (\mathbf{b} - \mathbf{x})' \Gamma^{-1} (\mathbf{b} - \mathbf{x}) \quad (1)$$

Denoting by $\boldsymbol{\omega}$ the information set of the farm at the beginning of the cropping season, we define $\alpha \equiv E[a | \boldsymbol{\omega}]$ and $\boldsymbol{\beta} \equiv E[\mathbf{b} | \boldsymbol{\omega}]$, $\varepsilon^\alpha \equiv a - \alpha$ and $\boldsymbol{\varepsilon}^\beta \equiv \mathbf{b} - \boldsymbol{\beta}$. The terms ε^α and $\boldsymbol{\varepsilon}^\beta$ are random from the viewpoint of the farmer at the start of the cropping season. Note that the terms α and $\boldsymbol{\beta}$ generally depend on farm’s natural assets as well as past production choices due to rotation effects, *i.e.* due to dynamic features of the crop production process. Farmer’s information set $\boldsymbol{\omega}$ is assumed to include \mathbf{w} and p .

We further partition the set of variable inputs into two subsets with $\mathbf{x} \equiv (\mathbf{x}_0, \mathbf{x}_1)$. Input quantities \mathbf{x}_0 are chosen at the start of the cropping season and cannot be adapted during the cropping season. The terms $\boldsymbol{\beta}$ and $\boldsymbol{\varepsilon}^\beta$ are partitioned accordingly with $\boldsymbol{\beta} \equiv (\boldsymbol{\beta}_0, \boldsymbol{\beta}_1)$ and $\boldsymbol{\varepsilon}^\beta \equiv (\boldsymbol{\varepsilon}_0^\beta, \boldsymbol{\varepsilon}_1^\beta)$. The $\boldsymbol{\varepsilon}_1^\beta$ random effects are observed during the production process and can be used to adapt \mathbf{x}_1 accordingly. We further assume that $\boldsymbol{\varepsilon}_0^\beta \equiv \mathbf{0}$. These conditions merely define the α , $\boldsymbol{\beta}$, ε^α and $\boldsymbol{\varepsilon}_1^\beta$ terms, and their interpretations. The term $\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta$ defines the effects of the factors affecting the crop production process which can be controlled using inputs 1. The term $\boldsymbol{\beta}_0$ defines the effects of the factors affecting the crop production process which can be controlled by using inputs 0. The term $\alpha + \varepsilon^\alpha$ defines the effects of the factors affecting the crop production process which are entirely undergone by the farmer. Note that if $E[\varepsilon^\alpha | \boldsymbol{\omega}, \boldsymbol{\varepsilon}_1^\beta] = 0$, other conditional moments of ε^α may depend on \mathbf{x} . This does not affect the following analysis as long as the considered farmer is risk neutral.

The risk neutral farmer optimal choice of inputs can be determined in two steps according to a standard backward induction approach. The corresponding steps follow the following decomposition:

$$Max_{\mathbf{x}_0 \geq 0} E \left[Max_{\mathbf{x}_1(\cdot) \geq 0} \left[py^e(\mathbf{x}_0, \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta), \boldsymbol{\varepsilon}_1^\beta) - \mathbf{w}'_1 \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta) \right] - \mathbf{w}'_0 \mathbf{x}_0 \right] \quad (2a)$$

of the farmer's program:

$$Max_{\mathbf{x}_0 \geq 0, \mathbf{x}_1(\cdot) \geq 0} \left[pE \left[y^e(\mathbf{x}_0, \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta), \boldsymbol{\varepsilon}_1^\beta) \right] - \mathbf{w}'_0 \mathbf{x}_0 - \mathbf{w}'_1 \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta) \right] \quad (2b)$$

where $y^e(\mathbf{x}, \boldsymbol{\varepsilon}_1^\beta)$ is the expected yield given \mathbf{x} and $\boldsymbol{\varepsilon}_1^\beta$

The first step corresponds to the choice of \mathbf{x}_1 provided that \mathbf{x}_0 has been chosen and that $\boldsymbol{\varepsilon}_1^\beta$ has been observed. The farmer solves the following problem:

$$Max_{\mathbf{x}_1 \geq 0} \left[py^e(\mathbf{x}, \boldsymbol{\varepsilon}_1^\beta) - \mathbf{w}'_1 \mathbf{x}_1 \right] \quad (3)$$

with:

$$y^e(\mathbf{x}, \boldsymbol{\varepsilon}_1^\beta) = \alpha - 1/2 \times (\boldsymbol{\beta}_0 - \mathbf{x}_0)' \mathbf{G}_{00} (\boldsymbol{\beta}_0 - \mathbf{x}_0) - (\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta - \mathbf{x}_1)' \mathbf{G}_{10} (\boldsymbol{\beta}_0 - \mathbf{x}_0) - 1/2 (\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta - \mathbf{x}_1)' \mathbf{G}_{11} (\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta - \mathbf{x}_1) \quad (4)$$

and:

$$\begin{bmatrix} \boldsymbol{\Gamma}_{00} & \boldsymbol{\Gamma}'_{10} \\ \boldsymbol{\Gamma}_{10} & \boldsymbol{\Gamma}_{11} \end{bmatrix} \equiv \boldsymbol{\Gamma} \quad \text{and} \quad \begin{bmatrix} \mathbf{G}_{00} & \mathbf{G}'_{10} \\ \mathbf{G}_{10} & \mathbf{G}_{11} \end{bmatrix} \equiv \boldsymbol{\Gamma}^{-1}. \quad (5)$$

This leads to the following optimal choice of \mathbf{x}_1 given \mathbf{x}_0 and $\boldsymbol{\varepsilon}_1^\beta$:

$$\mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta) = \boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta + \mathbf{G}_{11}^{-1} (\mathbf{G}_{10} (\boldsymbol{\beta}_0 - \mathbf{x}_0) - \mathbf{w}_1 p^{-1}) \quad (6)$$

which in turn implies that:

$$y^e(\mathbf{x}_0, \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta), \boldsymbol{\varepsilon}_1^\beta) = \alpha - 1/2 \times (\boldsymbol{\beta}_0 - \mathbf{x}_0)' (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10}) (\boldsymbol{\beta}_0 - \mathbf{x}_0) - 1/2 \times \mathbf{w}'_1 p^{-1} \mathbf{G}_{11}^{-1} \mathbf{w}_1 p^{-1} \quad (7)$$

When choosing \mathbf{x}_0 the farmer solves:

$$\text{Max}_{\mathbf{x}_0 \geq 0} \left[pE \left[y^e(\mathbf{x}_0, \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta), \boldsymbol{\varepsilon}_1^\beta) \right] - \mathbf{w}'_0 \mathbf{x}_0 - \mathbf{w}'_1 \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta) \right] \quad (8)$$

Solving this second step problem leads to the following optimal choice of \mathbf{x} :

$$\mathbf{x}_0(p, \mathbf{w}) = \boldsymbol{\beta}_0 + (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} (\mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{w}_1 p^{-1} - \mathbf{w}_0 p^{-1}) \quad (9a)$$

and:

$$\mathbf{x}_1(p, \mathbf{w}, \boldsymbol{\varepsilon}_1^\beta) = \boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta + \mathbf{G}_{11}^{-1} \mathbf{G}_{10} (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} \mathbf{w}_0 p^{-1} - \left[\mathbf{G}_{11}^{-1} + \mathbf{G}_{11}^{-1} \mathbf{G}_{10} (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \right] \mathbf{w}_1 p^{-1}. \quad (9b)$$

With:

$$\boldsymbol{\Gamma} = \begin{bmatrix} (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} & -(\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \\ -\mathbf{G}_{11}^{-1} \mathbf{G}_{10} (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} & \mathbf{G}_{11}^{-1} + \mathbf{G}_{11}^{-1} \mathbf{G}_{10} (\mathbf{G}_{00} - \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \mathbf{G}_{10})^{-1} \mathbf{G}'_{10} \mathbf{G}_{11}^{-1} \end{bmatrix}, \quad (10)$$

we have:

$$\mathbf{x}_0(p, \mathbf{w}) = \boldsymbol{\beta}_0 - \boldsymbol{\Gamma}_{00} \mathbf{w}_0 p^{-1} - \boldsymbol{\Gamma}'_{10} \mathbf{w}_1 p^{-1}, \quad (11a)$$

$$\mathbf{x}_1(p, \mathbf{w}, \boldsymbol{\varepsilon}_1^\beta) = \boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1^\beta - \boldsymbol{\Gamma}_{10} \mathbf{w}_0 p^{-1} - \boldsymbol{\Gamma}_{11} \mathbf{w}_1 p^{-1} \quad (11b)$$

and thus:

$$\mathbf{x}(p, \mathbf{w}, \boldsymbol{\varepsilon}^\beta) = \boldsymbol{\beta} + \boldsymbol{\varepsilon}^\beta - \boldsymbol{\Gamma} \mathbf{w} p^{-1}. \quad (11c)$$

Note that $y^e(\mathbf{x}_0, \mathbf{x}_1(\mathbf{x}_0, \boldsymbol{\varepsilon}_1^\beta), \boldsymbol{\varepsilon}_1^\beta)$ does not depend on $\boldsymbol{\varepsilon}_1^\beta$ thanks to the optimal “informed” choice of \mathbf{x}_1 . These results remain valid when p is only known at harvesting. It suffices to replace p^{-1} by its expectation at the beginning of the cropping season.

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